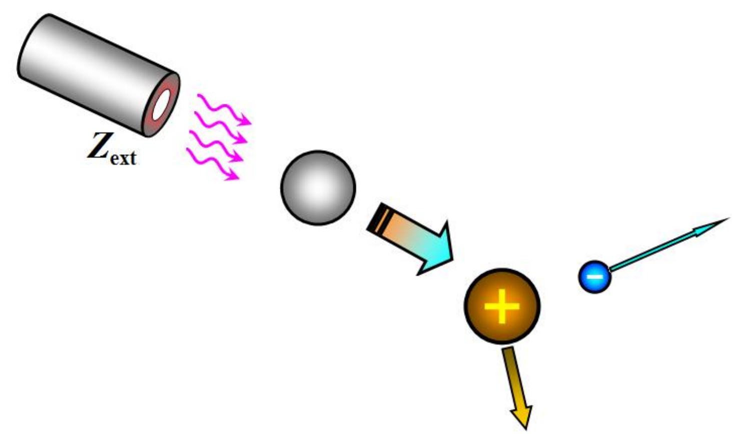
Problem 3. Simplest model of gas discharge

- This problem is a tribute to my teacher in science, Prof. F. Baimbetov

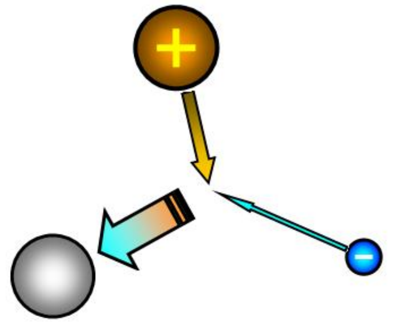
Part A. Non self-sustained gas discharge

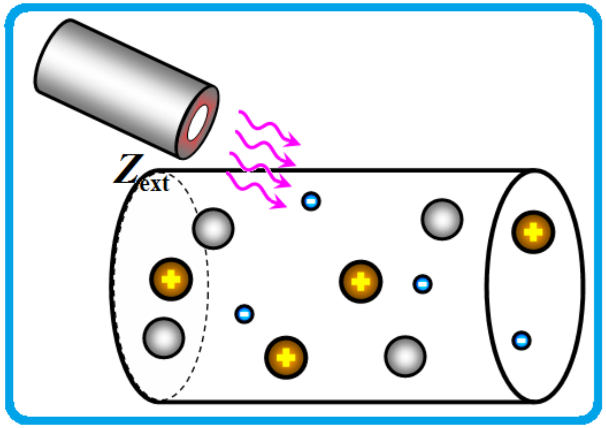
An external ionizer creates Z_{ext} pairs of singly ionized ions and free electrons per unit volume and per unit time.



The number of recombining events Z_{rec} that occurs in the gas per unit volume and per unit time is given by

$$Z_{rec} = r n_e n_i$$





Question A1

At time $t = 0$ the external ionizer is switched on and the initial number densities of electrons and ions in the gas are both equal to zero. The electron number density $n_e(t)$ depends on time t as follows:

$$n_e(t) = n_0 + a \tanh bt$$

Find n_0, a, b and express them in terms of Z_{ext} and r.

$$n(t) = n_e(t) = n_i(t)$$

$$\frac{dn(t)}{dt} = Z_{ext} - rn(t)^2$$

$$n_e(t) = n_0 + a \tanh bt$$

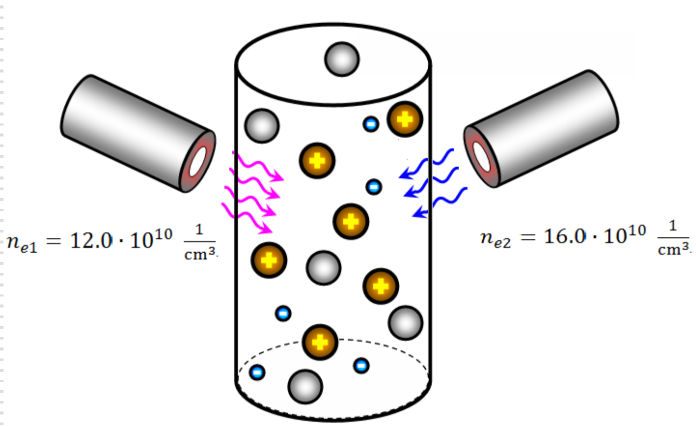
$t \to 0$ $\quad$ $\tanh bt \to 0$ $\quad$ $n_0 = 0$

$$n_e(t) = a \tanh bt$$

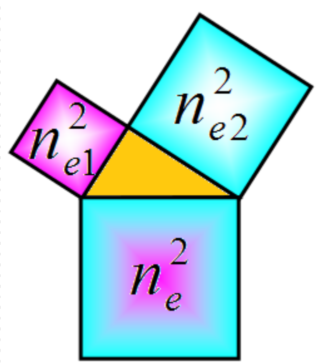
$$a = \sqrt{\frac{Z_{ext}}{r}}$$

$$b = \sqrt{rZ_{ext}}$$

Question A2



A2 Find the electron number density n_e at equilibrium when both external ionizers are switched on simultaneously.



$$\frac{dn(t)}{dt} = Z_{ext} - rn(t)^2$$

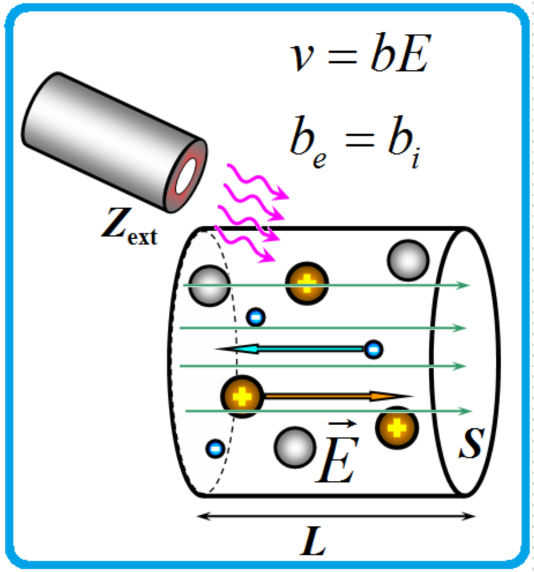
$$\bar{n} = \sqrt{\frac{Z_{ext}}{r}}$$

$$n_{e1} = \sqrt{\frac{Z_{ext1}}{r}} \qquad n_{e2} = \sqrt{\frac{Z_{ext2}}{r}} \qquad n_e = \sqrt{\frac{Z_{ext1}+Z_{ext2}}{r}}$$

$$n_e = \sqrt{n_{e1}^2 + n_{e2}^2} = 20.0 \cdot 10^{10}\mathrm{sm}^{-3}$$

Question A3

A3 Express the electric current I in the tube in terms of U, b, L, S, Z_{ext}, r.



$$Z_{ext}SL = rn_e n_i SL + \frac{I_e}{e}$$

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$I_e = I_i$

$I = I_e + I_i$

n_i, n_e

$$I_e = \frac{I}{2} = en_e vS = ebn_e ES$$

$$I_i = \frac{I}{2} = en_i vS = ebn_i ES$$

$$Z_{ext}SL = rSL\left(\frac{I}{2ebES}\right)^2 + \frac{I}{2e}$$

$$I = \frac{eb^2U^2S}{rL^3}\left(\sqrt{1 + \frac{4rZ_{ext}L^4}{b^2U^2}} - 1\right)$$

Question A4

A4 Find the resistivity ρ_{gas} of the gas at sufficiently small values of the voltage applied.

$$I = \frac{eb^2U^2S}{rL^3}\left(\sqrt{1+\frac{4rZ_{ext}L^4}{b^2U^2}}-1\right)$$

$$U \rightarrow 0$$

$$I = U2eb\sqrt{\frac{Z_{ext}}{r}}\frac{S}{L}$$

$$R = \frac{U}{I}$$

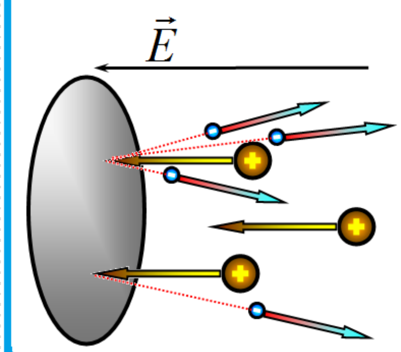
$$R = \rho\frac{L}{S}$$

$$\rho = \frac{1}{2eb}\sqrt{\frac{r}{Z_{ext}}}$$

Part B. Self-sustained gas discharge

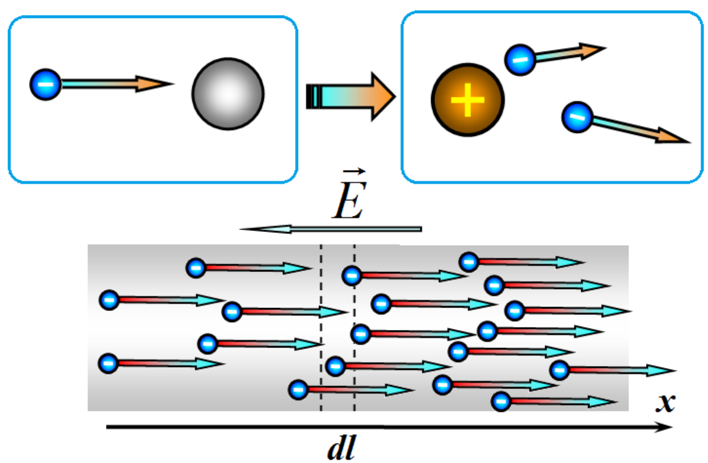
Attention! *In the sequel assume that the external ionizer continues to operate, neglect the electric field due to the charge carriers such that the electric field is uniform along the tube, and the recombination can be completely ignored.*

A secondary electron emission!



$$\gamma = \frac{\dot{N}_e}{\dot{N}_i}$$

A formation of electron avalanche!

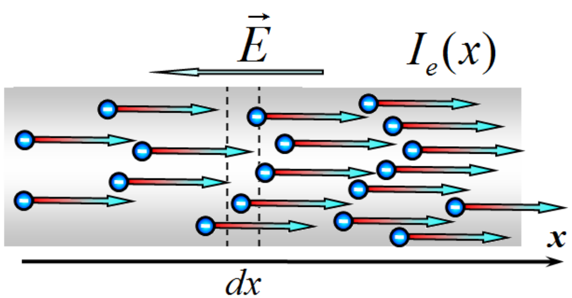


$$\frac{dN_e}{dl} = \alpha N_e$$

Question B1

$$I_e(x) = C_1 e^{A_1 x} + A_2$$

Find A_1, A_2 and express them in terms of Z_{ext}, α, e, L, S.



$$dN_e^I = \frac{I_e(x+dx) - I_e(x)}{e} = \frac{1}{e}\frac{dI_e(x)}{dx} dxdt$$

$$dN_e^{ext} = Z_{ext} S dxdt$$

$$dN_e^a = \alpha N_e dl = n_e S dx v dt = \alpha \frac{I_e(x)}{e} dxdt$$

$$dN_e^I = dN_e^{ext} + dN_e^a$$

$$\frac{dI_e(x)}{dx} = eZ_{ext}S + \alpha I_e(x)$$

$$I_e(x) = C_1 e^{A_1 x} + A_2$$

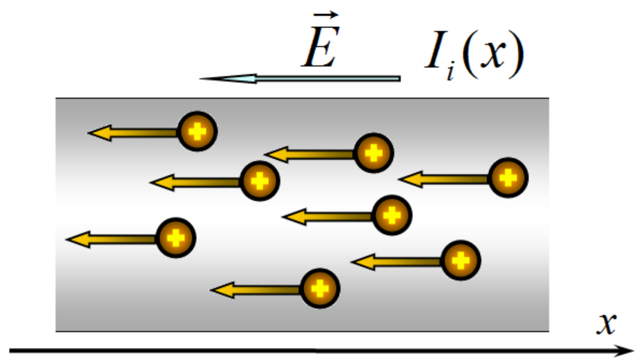
$$A_1 = \alpha$$

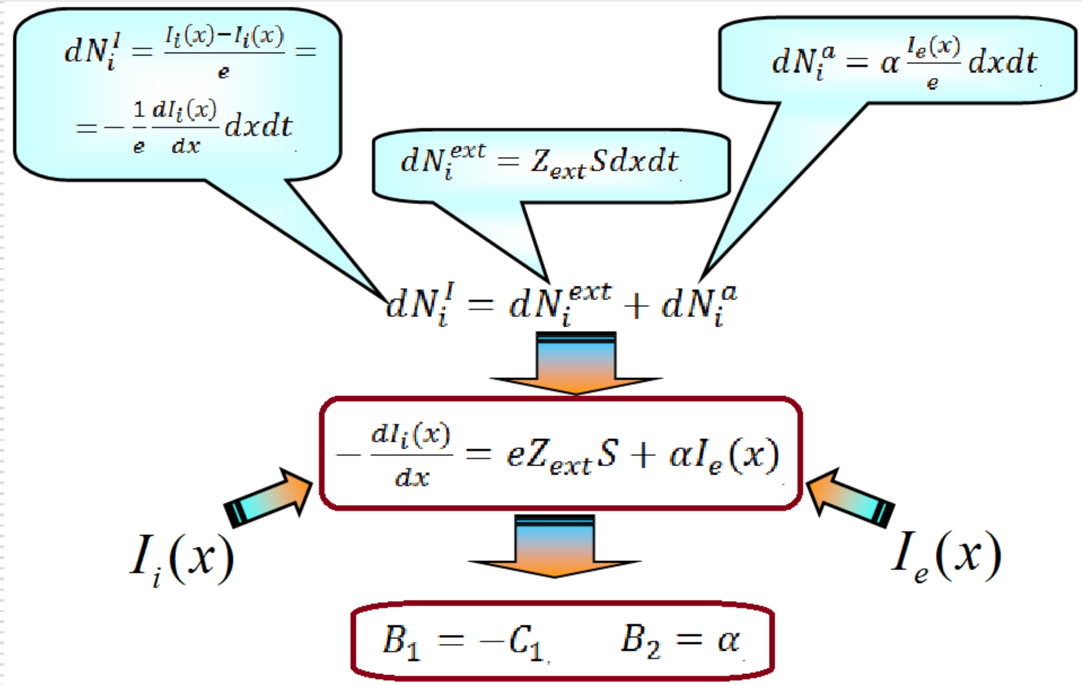
$$A_2 = -\frac{eZ_{ext}S}{\alpha}$$

Question B2

$$I_i(x) = C_2 + B_1 e^{B_2 x}$$

Find B_1, B_2 and express them in terms of $Z_{ext}, \alpha, e, L, S, \hat{C}_1$.



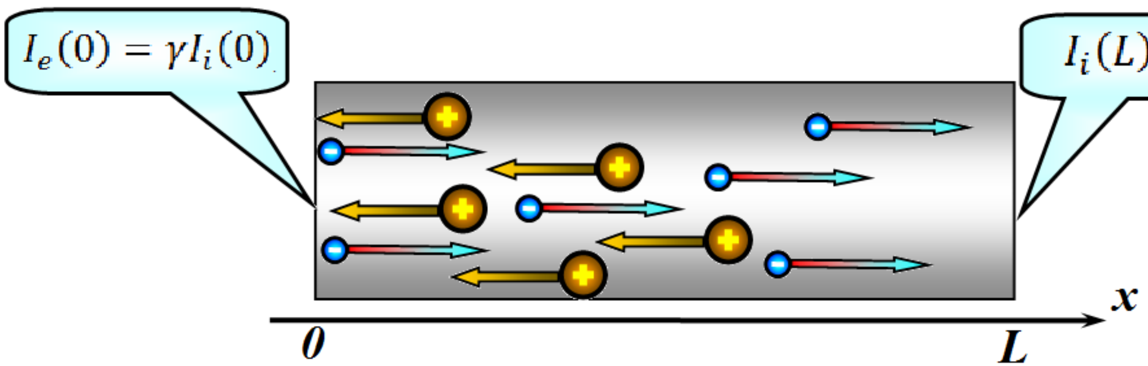


Question B3-B5

B3 Write down the condition for $I_i(x)$ at $x = L$.

B4 Write down the condition for $I_i(x)$ and $I_e(x)$ at $x = 0$.

B5 Find the total current I and express it in terms of $Z_{ext}, \alpha, \gamma, e, L, S$.



$$I_e(x) = C_1 e^{A_1 x} + A_2$$
$$I_i(x) = C_2 + B_1 e^{B_2 x}$$

$$I = I_e(x) + I_i(x) = \frac{e Z_{ext} S}{\alpha\left[e^{-\alpha L} - \frac{\gamma}{\gamma+1}\right]}$$

Questions B6,B7

Let the Townsend coefficient α be constant. When the length of the tube turns out greater than some critical value, i.e. $L > L_{cr}$, the external ionizer can be turned off and the discharge becomes self-sustained.

B6 Find L_{cr} and express it in terms of $Z_{ext}, \alpha, \gamma, e, L, S$.

$$\alpha = \alpha_1 p \exp\left(-\frac{\alpha_2 p}{E}\right),$$

B7 Find and evaluate the minimum voltage U_{min} applied across the plates, at which the self-sustained gas discharge can still appear in neon.

$$I = I_e(x) + I_i(x) = \frac{e Z_{ext} S}{\alpha\left[e^{-\alpha L} - \frac{\gamma}{\gamma+1}\right]}$$

$\left[e^{-\alpha L} - \frac{\gamma}{\gamma+1}\right] \to 0$

$$L_{cr} = \frac{1}{\alpha}\ln\left(1 + \frac{1}{\gamma}\right)$$

$$\alpha = \alpha_1 p \exp\left(-\frac{\alpha_2 p}{E}\right)$$

$U = EL$

$$U = \frac{\alpha_2 p L}{\ln\left(\frac{\alpha_1 p L}{\ln(1+1/\gamma)}\right)}$$

$(z = pL)$

$$\frac{dU}{dz} = 0$$

$$z_{min} = \frac{e}{\alpha_1}\ln(1 + 1/\gamma)$$

$$U_{min} = \frac{e\alpha_1}{\alpha_2}\ln\left(1 + \frac{1}{\gamma}\right) = 122\text{B}$$