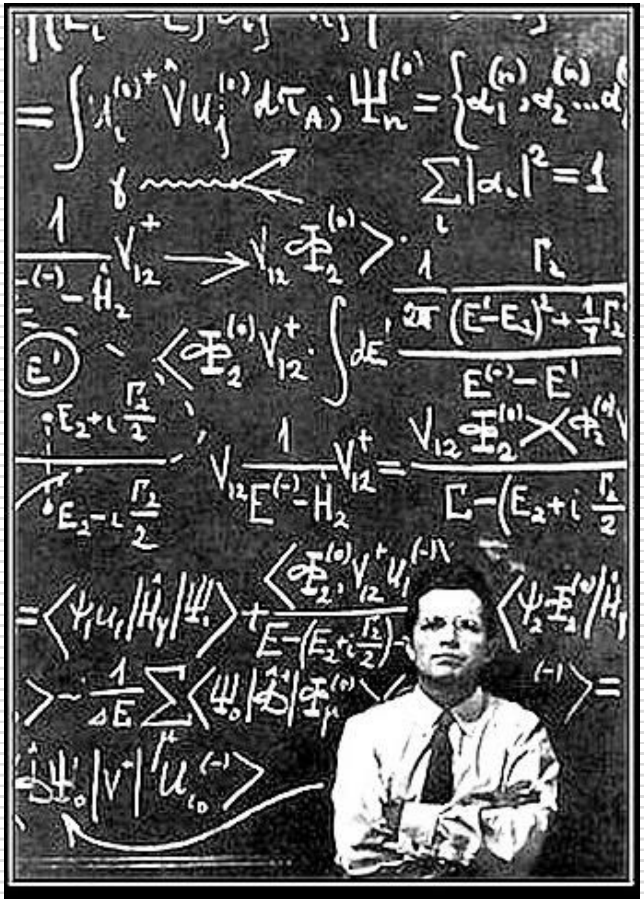
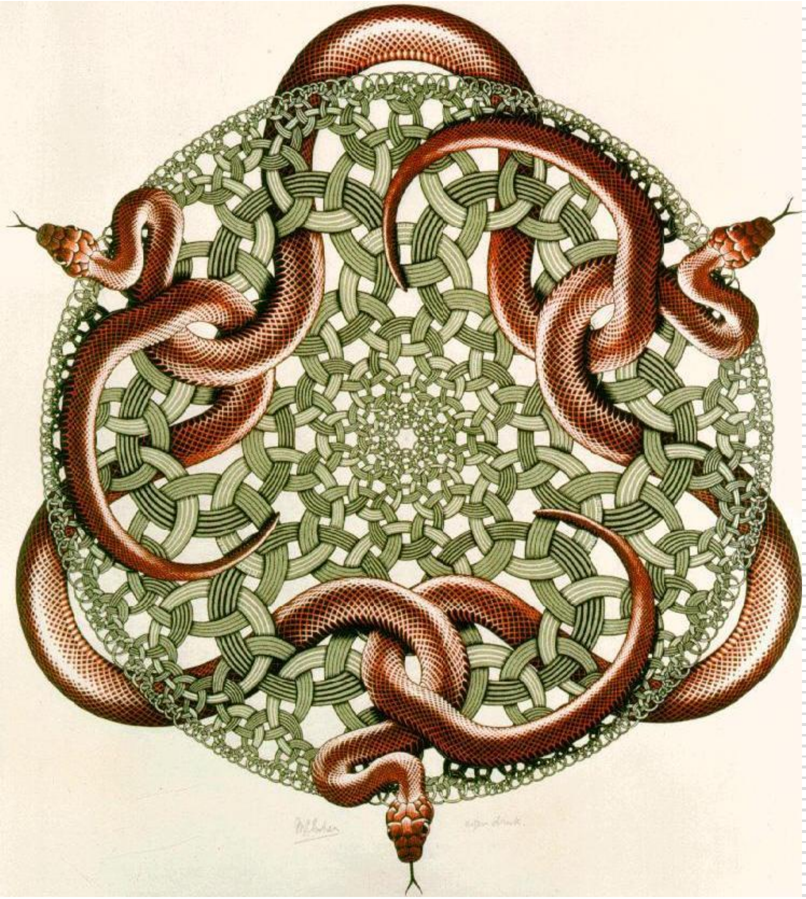
45th IPhO
2014 Kazakhstan-Astana

Theoretical competition 14.07.2014



- Problem 1 – 9 points
- Problem 2 – 12 points
- Problem 3 – 9 points

Problem 1



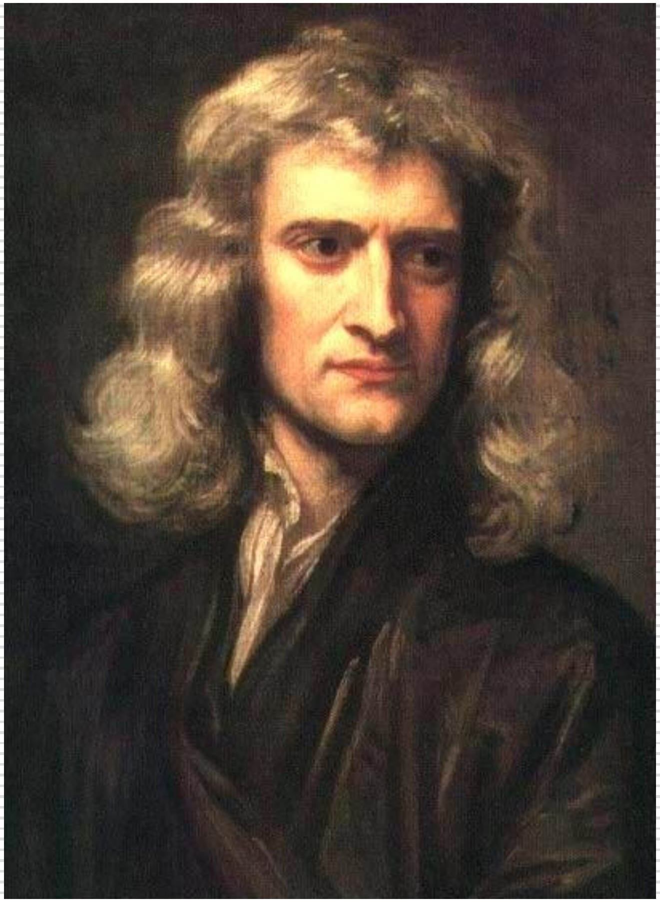
This problem consists of three independent parts:

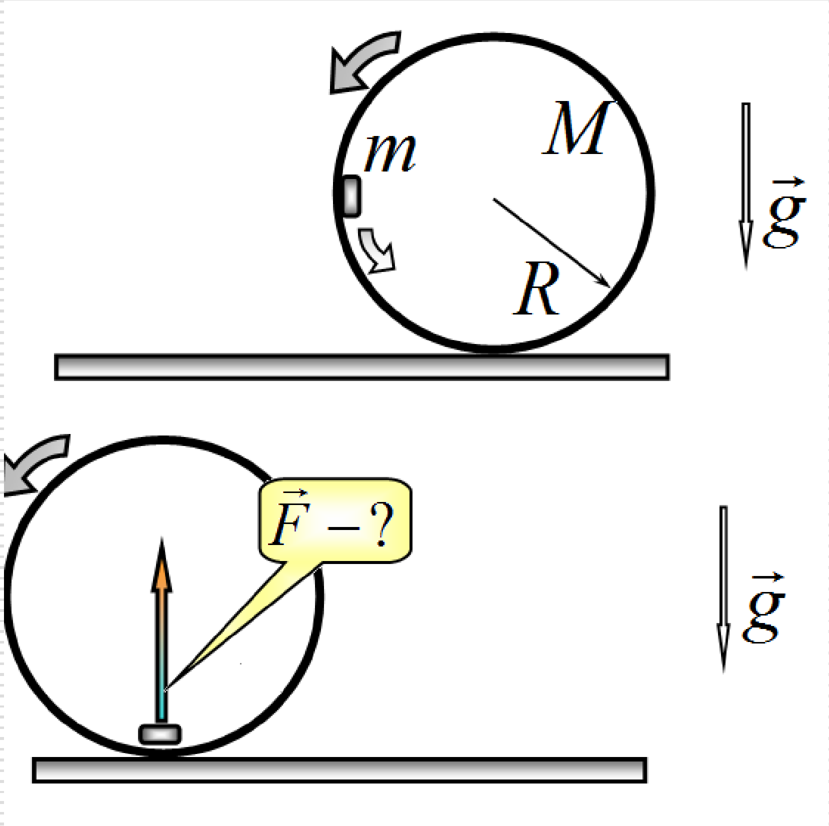
Part A – mechanics;

Part B – molecular physics;

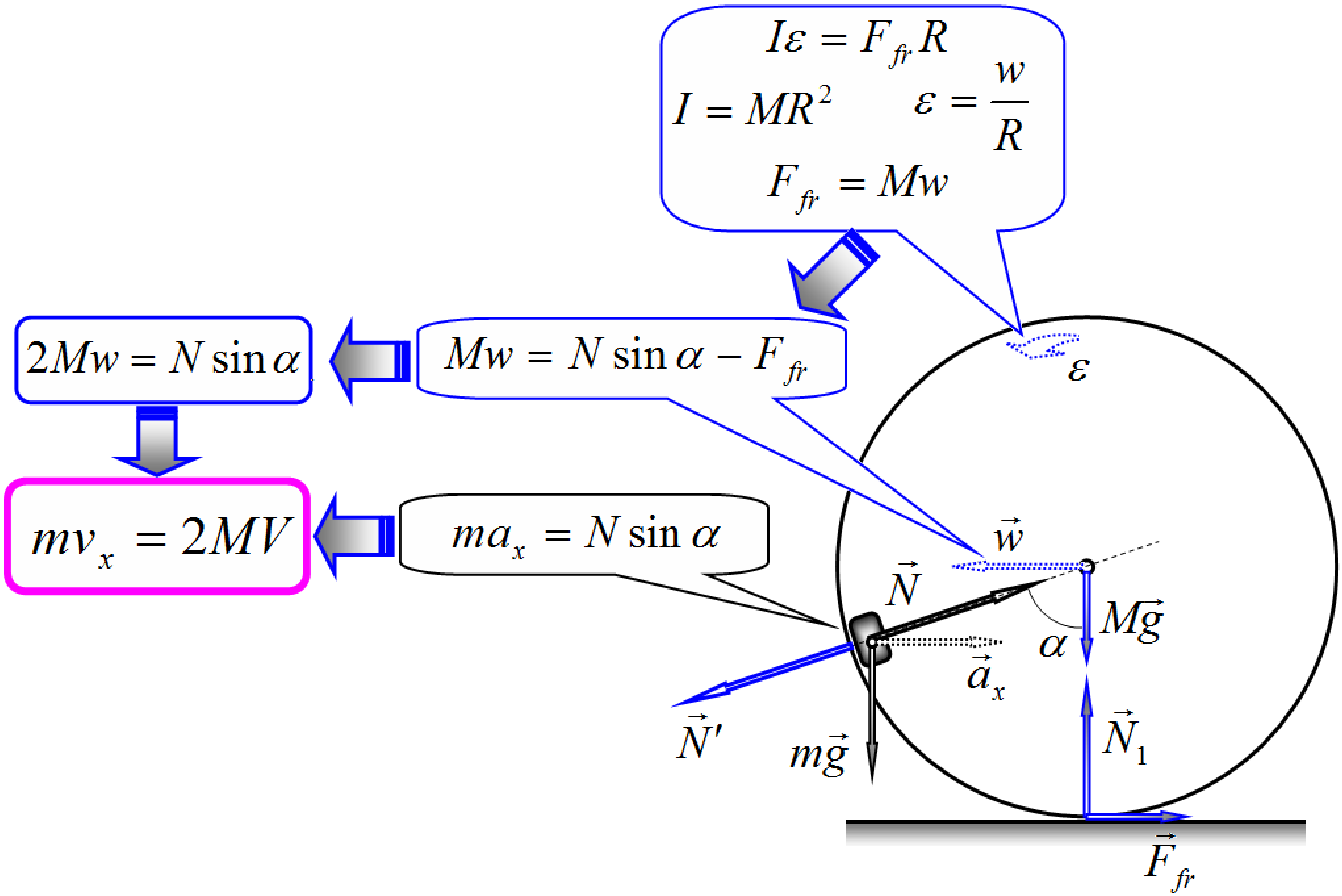
Part C – electricity.

Problem 1. Part A





Problem 1. Part A

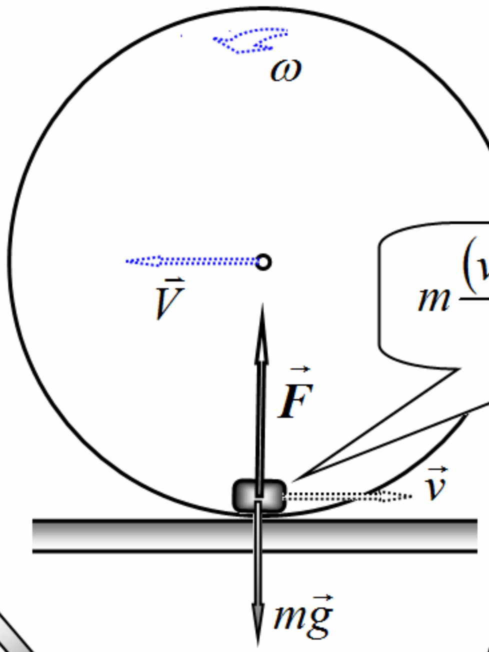


Problem 1. Part A

$$mv_x = 2MV$$

$$mgR = \frac{mv^2}{2} + \frac{MV^2}{2} + \frac{I\omega^2}{2} = \frac{mv^2}{2} + MV^2$$

$$v = \sqrt{\frac{2gR}{1+\frac{m}{2M}}} \qquad V = \frac{m}{2M}\sqrt{\frac{2gR}{1+\frac{m}{2M}}}$$



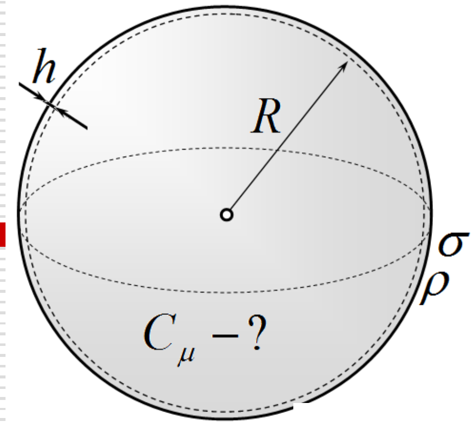
$$m\frac{(v+V)^2}{R} = \boldsymbol{F} - mg$$

$$N = mg\left(3+\frac{m}{M}\right)$$

Problem 1. Part B

- Competence in thermodynamics; equation for small oscillations

Question B1



$$\delta Q = \nu C_V dT + p dV$$

$$C = \frac{1}{\nu}\frac{\delta Q}{dT} = C_V + \frac{p}{\nu}\frac{dV}{dT}$$

$$C_V = \frac{5}{2}R$$

$$p = \frac{4\sigma}{r}$$

$$pV = \nu RT$$

$$p^3 V = const$$

$$T^3 V^{-2} = const$$

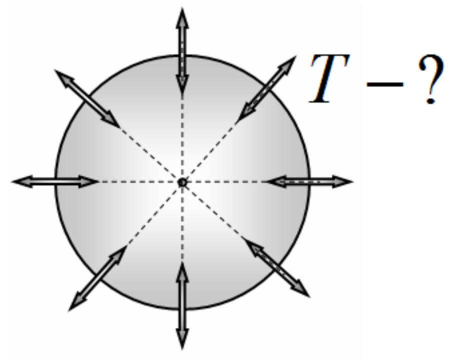
$$\frac{dV}{dT} = \frac{3V}{2T}$$

$$C = C_V + \frac{3}{2}R = 4R = 33.2\ \frac{\text{Дж}}{\text{моль}\cdot\text{К}}$$

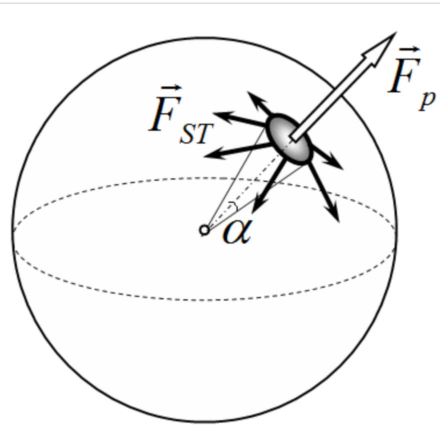
Alternative!

$$PV^{\frac{1}{3}} = const \qquad PV^n = const$$

$$n = \frac{C - C_P}{C - C_V} = \frac{1}{3} \Rightarrow C = 4R$$



Question B2



$$m\ddot{x} = p'S - F_{surf}$$

$$m = \rho S h$$
$$S = \pi(\alpha r)^2$$

$$F_{surf} = F_{ST}\alpha =$$
$$= \sigma \cdot 2 \cdot 2\pi[(r + x)\alpha] \cdot \alpha$$

T = const *(!)*

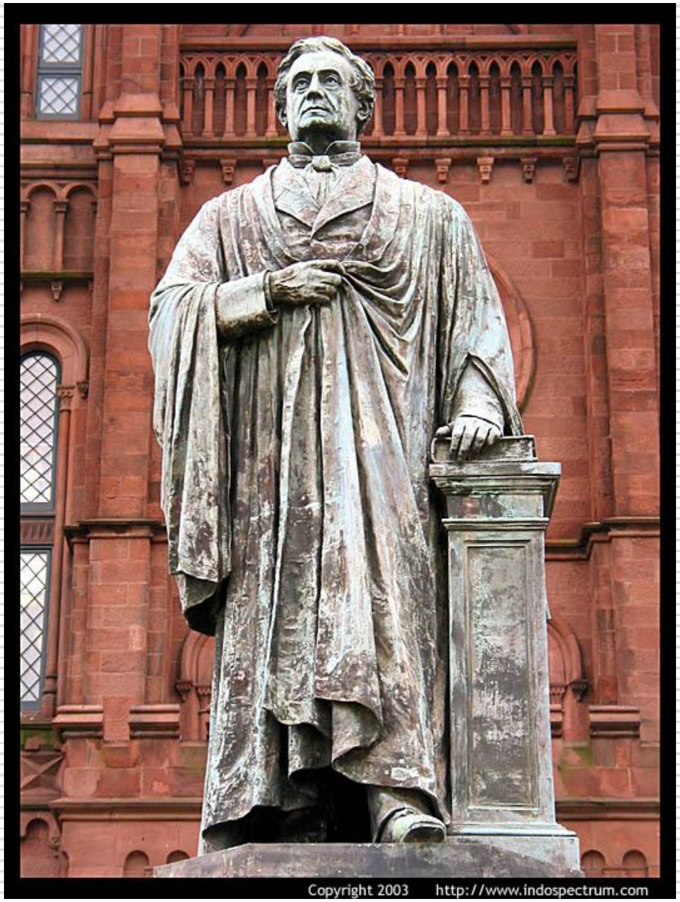
$$p'V' = pV$$

$$p' = p\frac{1}{\left(1+\frac{x}{r}\right)^3} \approx p\left(1 - \frac{3x}{r}\right)$$

$$\rho h\ddot{x} = -\frac{8\sigma}{r^2}x$$

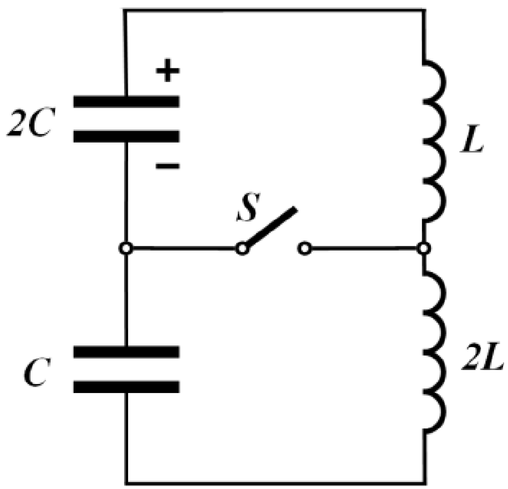
$$\omega = \sqrt{\frac{8\sigma}{\rho h r^2}} = 108\ c^{-1}$$

Problem 1. Part C



- Competence in LC circuits; vector diagrams

Problem 1. Part C



Initially, a switch is unshorted in the circuit shown in the figure on the right, a capacitor of capacitance $2C$ carries the electric charge q_0, a capacitor of capacitance C is uncharged, and there are no electric currents in both coils of inductance L and $2L$, respectively. The capacitor starts to discharge and at the moment when the current in the coils reaches its maximum value, the switch is instantly shorted. Find the maximum current I_{max} through the switch thereafter.

Problem 1. Part C

Method 1. Direct approach

At the moment when the current in the coils is a maximum, the total voltage across the coils is equal to zero, so the capacitor voltages must be equal in magnitude and opposite in polarity. Let U be a voltage on the capacitors at the time moment just mentioned and I_0 be that maximum current. According to the law of charge conservation

$$q_0 = 2CU + CU, \qquad (C2.1)$$

thus,

$$U = \frac{q_0}{3C}. \qquad (C2.2)$$

Then, from the energy conservation law

$$\frac{q_0^2}{2 \cdot 2C} = \frac{LI_0^2}{2} + \frac{2LI_0^2}{2} + \frac{CU^2}{2} + \frac{2CU^2}{2} \qquad (C2.3)$$

the maximum current is found as

$$I_0 = \frac{q_0}{3\sqrt{2LC}}. \qquad (C2.4)$$

After the key K is shortened there will be independent oscillations in both circuits with the frequency

$$\omega = \frac{1}{\sqrt{2LC}}, \qquad (C2.5)$$

and their amplitudes are obtained from the corresponding energy conservation laws written as

$$\frac{2CU^2}{2} + \frac{LI_0^2}{2} = \frac{LI_1^2}{2}, \qquad (C2.6)$$

$$\frac{CU^2}{2} + \frac{2LI_0^2}{2} = \frac{LI_2^2}{2}. \qquad (C2.7)$$

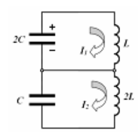
Hence, the corresponding amplitudes are found as

$$I_1 = \sqrt{5} I_0, \qquad (C2.8)$$

$$I_2 = \sqrt{2} I_0. \qquad (C2.9)$$

Choose the positive directions of the currents in the circuits as shown in the figure on the right. Then, the current flowing through the key is written as follows

$$I = I_1 \quad I_2. \qquad (C2.10)$$



The currents depend on time as

$$I_1(t) = A\cos\omega t + B\sin\omega t, \qquad (C2.11)$$

$$I_2(t) = D\cos\omega t + F\sin\omega t, \qquad (C2.12)$$

The constants A, B, D, F can be determined from the initial values of the currents and their amplitudes by putting down the following set of equations

$$I_1(0) = A = I_0, \qquad (C2.13)$$

$$A^2 + B^2 = I_1^2, \qquad (C2.14)$$

$$I_2(0) = D = I_0, \qquad (C2.15)$$

$$D^2 + F^2 = I_2^2. \qquad (C2.16)$$

Solving (C.13)-(C.16) it is found that

$$B = 2I_0, \qquad (C2.17)$$

$$F = -I_0, \qquad (C2.18)$$

The sign in F is chosen negative, since at the time moment of the key shortening the current in the coil $2L$ decreases.

Thus, the dependence of the currents on time takes the following form

$$I_1(t) = I_0(\cos\omega t + 2\sin\omega t), \qquad (C2.19)$$

$$I_2(t) = I_0(\cos\omega t - \sin\omega t). \qquad (C2.20)$$

In accordance with (C.10), the current in the key is dependent on time according to

$$I(t) = I_1(t) - I_2(t) = 3I_0\cos\omega t. \qquad (C2.21)$$

Hence, the amplitude of the current in the key is obtained as

$$I_{max} = 3I_0 = \omega q_0 = \frac{q_0}{\sqrt{2LC}} \qquad (C2.22)$$

$$I(t) = I_1(t) - I_2(t) = 3I_0\cos\omega t. \qquad (C2.21)$$

Hence, the amplitude of the current in the key is obtained as

$$I_{max} = 3I_0 = \omega q_0 = \frac{q_0}{\sqrt{2LC}} \qquad (C2.22)$$

$$I_{max} = \frac{q_0}{\sqrt{2LC}} \qquad \textbf{(C2.22)}$$

Problem 1. Part C

Method 2. Vector diagram

Instead of determining the coefficients A, B, D, F the vector diagram shown in the figure on the right can be used. The segment AC represents the current sought and its projection on the current axis is zero at the time of the key shortening. The current I_1 in the coil of inductance L grows at the same time moment because the capacitor $2C$ continues to discharge, thus, this current is depicted in the figure by the segment OC. The current I_2 in the coil of inductance $2L$ decreases at the time of the key shortening since it continues to charge the capacitor $2C$, that is why this current is depicted in the figure by the segment OA.

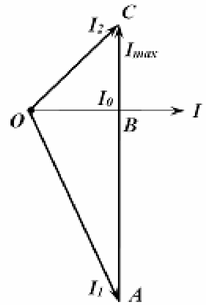
It is known for above that $OB = I_0, OA = \sqrt{5}I_0, OC = \sqrt{2}I_0$. Hence, it is found from the Pythagorean theorem that

$$AB = \sqrt{OA^2 - OB^2} = 2I_0, \qquad \text{(C3.1)}$$

$$BC = \sqrt{OC^2 - OB^2} = I_0, \qquad \text{(C3.2)}$$

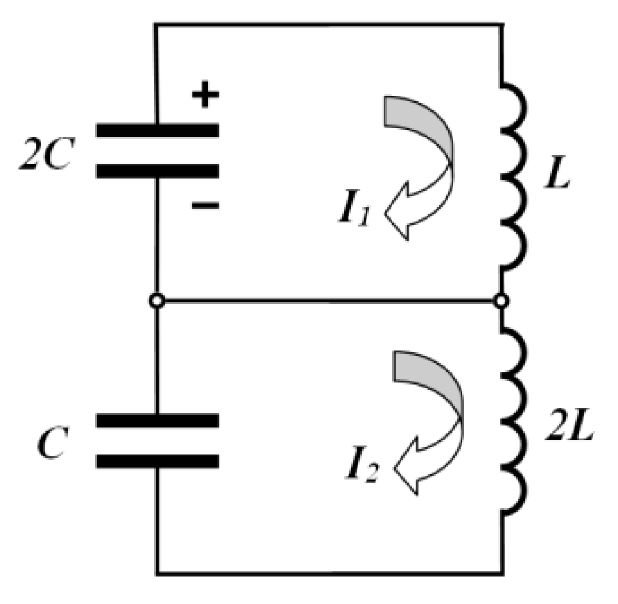
Thus, the current sought is found as

$$I_{max} = AC = AB + BC = 3I_0 = \omega q_0 = \frac{q_0}{\sqrt{2LC}}. \qquad \text{(C3.3)}$$



$$I_{max} = \frac{q_0}{\sqrt{2LC}}$$

Problem 1. Part C



Method 3. Heuristic approach

$$\omega = \frac{1}{\sqrt{2LC}}. \quad \text{(C1.1)}$$

$$I(t) = I_{\max} \sin \omega t. \quad \text{(C1.2)}$$

$$q_2 = q_0 - q_1. \quad \text{(C1.3)}$$

$$\dot{I}_1 = \frac{q_1}{2LC}, \quad \text{(C1.4)}$$

$$\dot{I}_2 = \frac{q_0 - q_1}{2LC}. \quad \text{(C1.5)}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = \frac{q_0}{2LC} = \omega^2 q_0. \quad \text{(C1.6)}$$

Note that this derivative is independent of the time of the key shortening!

$$I_{\max} = \frac{\dot{I}}{\omega} = \omega q_0 = \frac{q_0}{\sqrt{2LC}},$$