The 43rd International Physics Olympiad — Experimental Competition
Tartu, Estonia — Thursday, July 19th 2012

- The examination lasts for 5 hours. There are 2 problems worth in total 20 points. There are two tables in your disposal (in two neighbouring cubicles), the apparatus of Problem E1 is on one table and the apparatus of Problem E2 is on the other table; you can move freely between these tables. **However, you are not allowed to move any piece of experimental setup from one table to the other.**

- Initially the experimental equipment on one table is covered and on the other table is boxed. **You must neither remove the cover nor open the box nor open the envelope with the problems before the sound signal of the beginning of competition (three short signals).**

- **You are not allowed to leave your working place without permission.** If you need any assistance (**malfunctioning equipment**, broken calculator, need to visit a restroom, etc), please raise the corresponding flag (**“help”** or **“toilet”** with a long handle at your seat) above your seat box walls and keep it raised until an organizer arrives.

- Use only the front side of the sheets of paper.

- For each problem, there are **dedicated Solution Sheets** (see header for the number and pictogramme). Write your solutions onto the appropriate Solution Sheets. For each Problem, the Solution Sheets are numbered; use the sheets according to the enumeration. Copy the final answers into the appropriate boxes of the **Answer Sheets**.

- There are also **Draft papers**; use these for writing things which you don’t want to be graded. If you have written something what you don’t want to be graded onto the Solution Sheets (such as initial and incorrect solutions), cross these out.

- **If you need more paper for a certain problem, please raise the flag “help” and tell an organizer the problem number; you are given two Solution sheets (you can do this more than once).**

- **You should use as little text as possible:** try to explain your solution mainly with equations, numbers, tables, symbols and diagrams.

- Avoid unnecessary movements during the experimental examination and do not shake the walls of your cubicle - the laser experiment requires stability.

- **Do not look into the laser beam or its reflections! It may permanently damage your eyes!**

- The first single sound signal tells you that there are 30 min of solving time left; the second double sound signal means that 5 min is left; the third triple sound signal marks the end of solving time. **After the third sound signal you must stop writing immediately.** Put all the papers into the envelope at your desk. **You are not allowed to take any sheet of paper out of the room.** If you have finished solving before the final sound signal, please raise your flag.
Problem E1. The magnetic permeability of water (10 points)

The effect of a magnetic field on most of substances other than ferromagnetics is rather weak. This is because the energy density of the magnetic field in substances of relative magnetic permeability \( \mu \) is given by the formula \( w = \frac{B^2}{2\mu_0} \), and typically \( \mu \) is very close to 1. Still, with suitable experimental techniques such effects are firmly observable. In this problem we study the effect of a magnetic field, created by a permanent neodymium magnet, on water and use the results to calculate the magnetic permeability of water. You are not asked to estimate any uncertainties throughout this problem and you do not need to take into account the effects of surface tension.

The setup comprises of 1 a stand (the highlighted numbers correspond to the numbers in the fig.), 3 a digital caliper, 4 a laser pointer, 5 a water tray and 7 a cylindrical permanent magnet in the water tray (the magnet is axially magnetised). The water tray is fixed to the base of the stand by the magnet’s pull. The laser is fixed to the caliper, the base of which is fastened to the stand; the caliper allows horizontal displacement of the laser. The on-off button of the laser can be kept down with the help of 13 the white conical tube. Do not leave the Laser switched on unnecessarily. The depth of the water above the magnet should be reasonably close to 1 mm (if shallower, the water surface becomes so curved that it will be difficult to take readings from the screen). 15 A cup of water and 16 a syringe can be used for the water level adjustment (to raise the level by 1 mm, add 13 ml of water). 2 A sheet of graph paper (the “screen”) is to be fixed to the vertical plate with 14 small magnetic tablets. If the laser spot on the screen becomes smeared, check for a dust on the water surface (and blow away).

The remaining legend for the figure is as follows: 6 the point where the laser beam hits the screen; 11 the LCD screen of the caliper, 10 the button which switches the caliper units between millimeters and inches; 8 on-off switch; 9 button for setting the origin of the caliper reading. Beneath the laser pointer, there is one more button on the caliper, which temporarily re-sets the origin (if you pushed it inadvertently, push it once again to return to the normal measuring mode).

Numerical values for your calculations:

- Horizontal distance between the magnet’s centre and the screen \( L_0 = 490 \) mm. Check (and adjust, if needed) the alignment of the centre of the magnet in two perpendicular directions. The vertical axis of the magnet must intersect with the laser beam, and it must also intersect with 12 the black line on the support plate.
- Magnetic induction (magnetic field strength) on the magnet’s axis, at a height of 1 mm from the flat surface, \( B_0 = 0.50 \) T
- Density of water \( \rho_w = 1000 \) kg/m\(^3\)
- Acceleration of free-fall \( g = 9.8 \) m/s\(^2\)
- Permeability of a vacuum \( \mu_0 = 4\pi \times 10^{-7} \) H/m

WARNINGS:
- The laser orientation is pre-adjusted, do not move it!
- Do not look into the laser beam or its reflections!
- Do not try to remove the strong neodymium magnet!
- Do not put magnetic materials close to the magnet!
- Turn off the laser when not used, batteries drain in 1 h!
Part A. Qualitative shape of the water surface (1 points)
When a cylindrical magnet is placed below water surface, the latter becomes curved. By observation, determine the shape of the water surface above the magnet. Based on this observation, decide if the water is diamagnetic ($\mu < 1$) or paramagnetic ($\mu > 1$).

Write the letter corresponding to the correct option into the Answer Sheet, together with an inequality $\mu > 1$ or $\mu < 1$.

For this part, you do not need to justify your answer.

Part B. Exact shape of the water surface (7 points)
Curving of the water surface can be checked with high sensitivity by measuring the reflection of the laser beam from the surface. We use this effect to calculate the dependence of the depth of the water on the horizontal position above the magnet.

i. (1.6 pts) Measure the dependence of the vertical position $y$ of the laser spot on the screen on the caliper reading $x$ (see figure). You should cover the whole usable range of caliper displacements. Write the results into the Table in the Answer Sheet.

ii. (0.7 pts) Draw the graph of the measured dependence.

iii. (0.7 pts) Using the obtained graph, determine the angle $\alpha_0$ between the beam and the horizontal surface of the water.

iv. (1.4 pts) please note that the slope $(\tan \beta)$ of the water surface can be expressed as follows:

$$
\tan \beta \approx \frac{\cos^2 \alpha_0}{2} \frac{y - y_0 - (x - x_0) \tan \alpha_0}{L_0 + x - x_0},
$$

where $y_0$ is the vertical position of the laser spot on the screen when the beam is reflected from the water surface at the axis of the magnet, and $x_0$ is the respective position of the caliper. Calculate the values of the slope of the water surface and enter them into the Table on the Answer Sheet. Please note that it may be possible to simplify your calculations if you substitute some combination of terms in the given expression for the slope with a reading from the last graph.

v. (1.6 pts) Calculate the height of the water surface relative to the surface far from the magnet, as a function of $x$, and write it into the Table on the Answer Sheet.

vi. (1.0 pts) Draw the graph of the latter dependence. Indicate on it the region where the beam hits the water surface directly above the magnet.

Part C. Magnetic permeability (2 points)
Using the results of Part B, calculate the value of $\mu - 1$ (the so-called magnetic susceptibility), where $\mu$ is the relative magnetic permeability of the water. Write your final formula and the numerical result into the Answer Sheet.
Problem E2. Nonlinear Black Box (10 points)

In simple problems, electrical circuits are assumed to consist of linear elements, for which electrical characteristics are directly proportional to each other. Examples include resistance (\( V = RI \)), capacitance (\( Q = CV \)) and inductance (\( V = LI \)), where \( R \), \( C \) and \( L \) are constants. In this problem, however, we examine a circuit containing nonlinear elements, enclosed in a black box, for which the assumption of proportionality no longer holds.

The setup comprises a multimeter (labelled “IPhO-measure”), a black box that acts as a current source, a black box containing nonlinear elements, and four test leads with stackable connectors for wiring. Be careful not to break the seal on the black box.

The multimeter can measure current and voltage simultaneously. You can store it up to 2000 data points, each consisting of: voltage \( V \), current \( I \), power \( P = IV \), resistance \( R = V/I \), voltage time derivative \( \frac{dV}{dt} \), current time derivative \( \frac{dI}{dt} \) and time \( t \). See multimeter manual for details. If you go beyond 2000 stored data points, the oldest data will be overwritten.

The constant current source supplies stable current as long as the voltage across its terminals stays between \(-0.6125\ V\) and \(0.6125\ V\). When switched off, the constant current source behaves as a large (essentially infinite) resistance.

The black box contains an electric double layer capacitor (which is a slightly nonlinear high capacitance capacitor), an unknown nonlinear element, and an inductor \( L = 10\ \mu\text{H} \) of negligible resistance, switchable as indicated on the circuit diagram. The nonlinear element can be considered as a resistance with a nonlinear dependence between the voltage and the current \( I(V) \) is a continuous function of \( V \) with \( I(0) = 0 \). Likewise, for the capacitor, the differential capacitance \( C(V) = \frac{dQ}{dV} \) is not exactly constant.

We say that the voltage on the black box is positive when the potential on its red terminal is higher than the potential on the black terminal. Positive voltage will be acquired when the terminals of matching col-ours on the black box and the current source are connected (you are allowed to use negative voltages).

It is safe to discharge the capacitor in the black box by shorting its inputs, either by itself or through the \( IN \) and \( OUT \) terminals on multimeter: the internal resistance of this capacitor is enough to keep the current from damaging anything.

You are not asked to estimate any uncertainties throughout this problem.

Part A. Circuit without inductance (7 points)

In this part, keep the switch on the black box closed (push “I” down), so that the inductance is shorted. Please note that some measurements may take a considerable time, therefore it is recommended that you read through all the tasks of part A to avoid unnecessary work.

i. (1.0 pts) Confirm that the output current of the current source is approximately \( 6\ \text{mA} \), and determine the range within which it varies for voltages between \( 0 \) and \( +480\ \text{mV} \). Document the circuit diagram used.

ii. (1.2 pts) Show that the differential capacitance \( C(V) \) used in the black box is approximately \( 2\ \text{F} \) by measuring its value for a single voltage of your choice \( C(V_0) = C_0 \). Document the circuit diagram.

iii. (2.2 pts) Neglecting the nonlinearity of the capacitance \( [C(V) \approx C_0] \), determine the current–voltage characteristic of the nonlinear element used in the black box. Plot the \( I(V) \) curve for obtainable positive voltages on the black box onto the answer sheet. Document the circuit diagram.

iv. (2.6 pts) Using measurements taken from the whole range of obtainable voltages, calculate and plot the \( C(V) \) curve for obtainable positive voltages from the black box on the answer sheet. Write down the minimal and maximal values of differential capacitance \( C_{\text{min}}, C_{\text{max}} \). Document the circuit diagram.

Part B. Circuit with inductance (3 points)

Enable the inductance by opening the switch on the black box (push “0” down). Using the same method as in pt. A-iii, measure and plot the current–voltage characteristic of the nonlinear element. Describe any significant differences between the curves of parts A and B and suggest a reason using qualitative arguments. You need to know that the nonlinear element also has a capacitance \( \approx 1\ \text{nF} \) which is connected in parallel to the nonlinear resistance.
**IPhO-measure: short manual**

*IPhO-measure* is a multimeter capable of measuring voltage \( V \) and current \( I \) simultaneously. It also records their time derivatives \( \dot{V} \) and \( \dot{I} \), their product \( P = VI \), ratio \( R = V/I \), and time \( t \) of the sample. Stored measurements are organized into separate sets; every stored sample is numbered by the set number \( s \) and a counter \( n \) inside the set. All saved samples are written to an internal flash memory and can later be retrieved.

**Electrical behaviour**

The device behaves as an ammeter and a voltmeter connected as follows.

![Multimeter diagram]

<table>
<thead>
<tr>
<th>Mode</th>
<th>Range</th>
<th>Internal resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltmeter</td>
<td>0...2 V</td>
<td>1 MΩ</td>
</tr>
<tr>
<td>Voltmeter</td>
<td>2...10 V</td>
<td>57 kΩ</td>
</tr>
<tr>
<td>Ammeter</td>
<td>0...1 A</td>
<td>1 Ω</td>
</tr>
</tbody>
</table>

**Basic usage**

- Push “Power” to switch the *IPhO-measure* on. The device is not yet measuring; to start measuring, push “Start”. Alternatively, you can now start browsing your stored data. See below.
- To browse previously saved samples (through all sets), press “Previous” or “Next”. Hold them down longer to jump directly between sets.
- While not measuring, push “Start” to start measuring a new set.
- While measuring, push “Sample” each time you want to store a new set of data (i.e. of the readings shown on the display).
- While measuring, you can also browse other samples of the current set, using “Previous” and “Next”.
- Press “Stop” to end a set and stop measuring. The device is still on. You are ready to start a new measuring session or start browsing stored data.
- Pushing “Power” turns the device off. The device will show text “my mind is going ...”; do not worry, all the data measurements will be stored and you will be able to browse them after you switch the device on again. Saved samples will not be erased.

**Display**

A displayed sample consists of nine variables:

1. index \( n \) of the sample in the set;
2. index \( s \) of the set;
3. time \( t \) since starting the set;
4. voltmeter output \( V \);
5. rate of change of \( V \) (the time derivative \( \dot{V} \)); if derivative cannot be reliably taken due to fluctuations, “+nan/s” is shown;
6. ammeter output \( I \);
7. rate of change of \( I \) (the time derivative \( \dot{I} \)); if derivative cannot be reliably taken due to fluctuations, “+nan/s” is shown;
8. product \( P = VI \);
9. ratio \( R = V/I \).

If any of the variables is out of its allowed range, its display shows “+inf” or “-inf”.