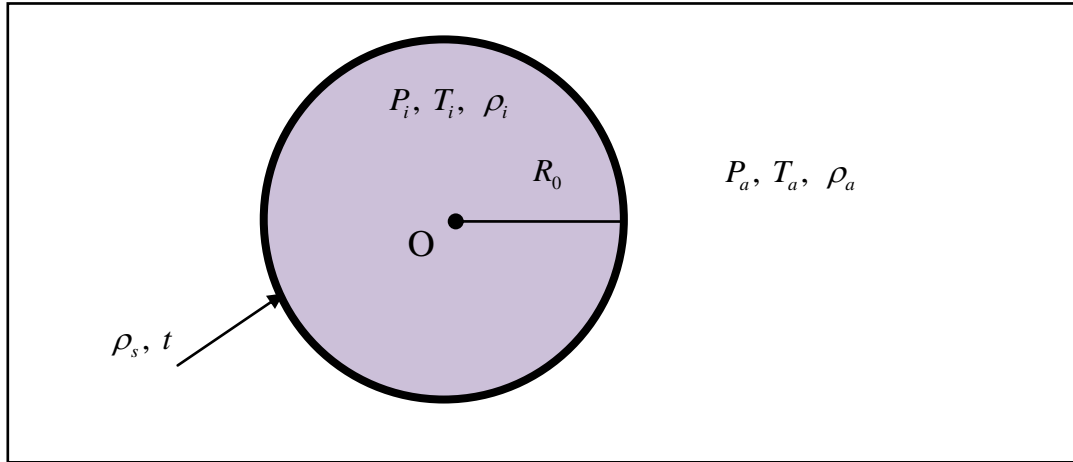


2. SOLUTION

2.1. The bubble is surrounded by air.



Cutting the sphere in half and using the projected area to balance the forces give

$$P_i \pi R_0^2 = P_a \pi R_0^2 + 2(2\pi R_0 \gamma) \quad \dots (1)$$

$$P_i = P_a + \frac{4\gamma}{R_0}$$

The pressure and density are related by the ideal gas law:

$$PV = nRT \quad \text{or} \quad P = \frac{\rho RT}{M}, \quad \text{where } M = \text{the molar mass of air.} \quad \dots (2)$$

Apply the ideal gas law to the air inside and outside the bubble, we get

$$\rho_i T_i = P_i \frac{M}{R}$$

$$\rho_a T_a = P_a \frac{M}{R},$$

$$\frac{\rho_i T_i}{\rho_a T_a} = \frac{P_i}{P_a} = \left[1 + \frac{4\gamma}{R_0 P_a} \right] \quad \dots (3)$$

- 2.2. Using $\gamma=0.025\text{Nm}^{-1}$, $R_0=1.0\text{ cm}$ and $P_a=1.013\times 10^5\text{ Nm}^{-2}$, the numerical value of the ratio is

$$\frac{\rho_i T_i}{\rho_a T_a} = 1 + \frac{4\gamma}{R_0 P_a} = 1 + 0.0001 \quad \dots (4)$$

(The effect of the surface tension is very small.)

- 2.3. Let W = total weight of the bubble, F = buoyant force due to air around the bubble

$$\begin{aligned} W &= (\text{mass of film} + \text{mass of air}) g \\ &= \left(4\pi R_0^2 \rho_s t + \frac{4}{3} \pi R_0^3 \rho_i \right) g \\ &= 4\pi R_0^2 \rho_s t g + \frac{4}{3} \pi R_0^3 \frac{\rho_a T_a}{T_i} \left[1 + \frac{4\gamma}{R_0 P_a} \right] g \end{aligned} \quad \dots (5)$$

The buoyant force due to air around the bubble is

$$B = \frac{4}{3} \pi R_0^3 \rho_a g \quad \dots (6)$$

If the bubble floats in still air,

$$\begin{aligned} B &\geq W \\ \frac{4}{3} \pi R_0^3 \rho_a g &\geq 4\pi R_0^2 \rho_s t g + \frac{4}{3} \pi R_0^3 \frac{\rho_a T_a}{T_i} \left[1 + \frac{4\gamma}{R_0 P_a} \right] g \end{aligned} \quad \dots (7)$$

Rearranging to give

$$\begin{aligned} T_i &\geq \frac{R_0 \rho_a T_a}{R_0 \rho_a - 3 \rho_s t} \left[1 + \frac{4\gamma}{R_0 P_a} \right] \\ &\geq 307.1 \text{ K} \end{aligned} \quad \dots (8)$$

The air inside must be about 7.1°C warmer.

- 2.4. Ignore the radius change \rightarrow Radius remains $R_0 = 1.0$ cm
(The radius actually decreases by 0.8% when the temperature decreases from 307.1 K to 300 K. The film itself also becomes slightly thicker.)

The drag force from Stokes' Law is $F = 6\pi\eta R_0 u$... (9)

If the bubble floats in the updraught,

$$F \geq W - B$$

$$6\pi\eta R_0 u \geq \left(4\pi R_0^2 \rho_s t + \frac{4}{3} \pi R_0^3 \rho_i \right) g - \frac{4}{3} \pi R_0^3 \rho_a g$$
 ... (10)

When the bubble is in thermal equilibrium $T_i = T_a$.

$$6\pi\eta R_0 u \geq \left(4\pi R_0^2 \rho_s t + \frac{4}{3} \pi R_0^3 \rho_a \left[1 + \frac{4\gamma}{R_0 P_a} \right] \right) g - \frac{4}{3} \pi R_0^3 \rho_a g$$

Rearranging to give

$$u \geq \frac{4R_0 \rho_s t g}{6\eta} + \frac{\frac{4}{3} R_0^2 \rho_a g \left(\frac{4\gamma}{R_0 P_a} \right)}{6\eta}$$
 ... (11)

- 2.5. The numerical value is $u \geq 0.36$ m/s.

The 2nd term is about 3 orders of magnitude lower than the 1st term.

From now on, ignore the surface tension terms.

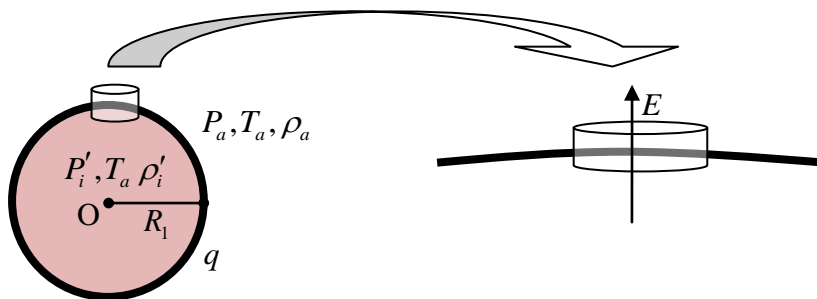
- 2.6. When the bubble is electrified, the electrical repulsion will cause the bubble to expand in size and thereby raise the buoyant force.

The force/area is (e-field on the surface \times charge/area)

There are two alternatives to calculate the electric field ON the surface of the soap film.

A. From Gauss's Law

Consider a very thin pill box on the soap surface.



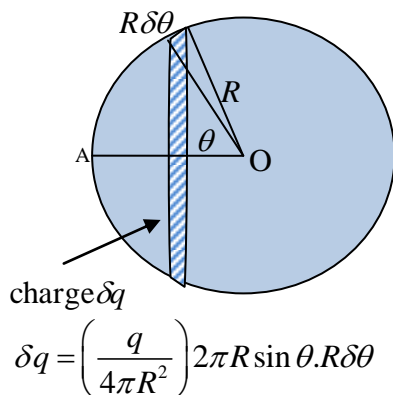
E = electric field on the film surface that results from all other parts of the soap film, excluding the surface inside the pill box itself.

$$\begin{aligned}
 E_q &= \text{total field just outside the pill box} = \frac{q}{4\pi\epsilon_0 R_1^2} = \frac{\sigma}{\epsilon_0} \\
 &= E + \text{electric field from surface charge } \sigma \\
 &= E + E_\sigma
 \end{aligned}$$

Using Gauss's Law on the pill box, we have $E_\sigma = \frac{\sigma}{2\epsilon_0}$ perpendicular to the film as a result of symmetry.

$$\text{Therefore, } E = E_q - E_\sigma = \frac{\sigma}{\epsilon_0} - \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0} = \frac{1}{2\epsilon_0} \frac{q}{4\pi R_1^2} \quad \dots (12)$$

B. From direct integration



To find the magnitude of the electrical repulsion we must first find the electric field intensity E at a point on (not outside) the surface itself.

Field at A in the direction \overrightarrow{OA} is

$$\delta E_A = \frac{1}{4\pi\epsilon_0} \frac{(q/4\pi R_1^2) 2\pi R_1^2 \sin\theta \delta\theta}{\left(2R_1 \sin\frac{\theta}{2}\right)^2} \sin\frac{\theta}{2} = \frac{(q/4\pi R_1^2)}{2\epsilon_0} \cos\frac{\theta}{2} \delta\left(\frac{\theta}{2}\right)$$

$$E_A = \frac{(q/4\pi R_1^2)}{2\epsilon_0} \int_{\theta=0}^{\theta=180^\circ} \cos\frac{\theta}{2} d\left(\frac{\theta}{2}\right) = \frac{(q/4\pi R_1^2)}{2\epsilon_0} \dots (13)$$

The repulsive force per unit area of the surface of bubble is

$$\left(\frac{q}{4\pi R_1^2}\right) E = \frac{(q/4\pi R_1^2)^2}{2\epsilon_0} \dots (14)$$

Let P'_i and ρ'_i be the new pressure and density when the bubble is electrified.

This electric repulsive force will augment the gaseous pressure P'_i .

P'_i is related to the original P_i through the gas law.

$$P'_i \frac{4}{3} \pi R_1^3 = P_i \frac{4}{3} \pi R_0^3$$

$$P'_i = \left(\frac{R_0}{R_1}\right)^3 P_i = \left(\frac{R_0}{R_1}\right)^3 P_a \dots (15)$$

In the last equation, the surface tension term has been ignored.

From balancing the forces on the half-sphere projected area, we have (again ignoring the surface tension term)

$$P'_i + \frac{(q/4\pi R_1^2)^2}{2\epsilon_0} = P_a \dots (16)$$

$$P_a \left(\frac{R_0}{R_1}\right)^3 + \frac{(q/4\pi R_1^2)^2}{2\epsilon_0} = P_a$$

Rearranging to get

$$\left(\frac{R_1}{R_0}\right)^4 - \left(\frac{R_1}{R_0}\right) - \frac{q^2}{32\pi^2 \varepsilon_0 R_0^4 P_a} = 0 \quad \dots (17)$$

Note that (17) yields $\frac{R_1}{R_0} = 1$ when $q = 0$, as expected.

2.7. Approximate solution for R_1 when $\frac{q^2}{32\pi^2 \varepsilon_0 R_0^4 P_a} \ll 1$

Write $R_1 = R_0 + \Delta R$, $\Delta R \ll R_0$

$$\text{Therefore, } \frac{R_1}{R_0} = 1 + \frac{\Delta R}{R_0}, \quad \left(\frac{R_1}{R_0}\right)^4 \approx 1 + 4\frac{\Delta R}{R_0} \quad \dots (18)$$

Eq. (17) gives:

$$\Delta R \approx \frac{q^2}{96\pi^2 \varepsilon_0 R_0^3 P_a} \quad \dots (19)$$

$$R_1 \approx R_0 + \frac{q^2}{96\pi^2 \varepsilon_0 R_0^3 P_a} \approx R_0 \left(1 + \frac{q^2}{96\pi^2 \varepsilon_0 R_0^4 P_a}\right) \quad \dots (20)$$

2.8. The bubble will float if

$$B \geq W$$

$$\frac{4}{3}\pi R_1^3 \rho_a g \geq 4\pi R_0^2 \rho_s t g + \frac{4}{3}\pi R_0^3 \rho_l g \quad \dots (21)$$

Initially, $T_i = T_a \Rightarrow \rho_i = \rho_a$ for $\gamma \rightarrow 0$ and $R_1 = R_0 \left(1 + \frac{\Delta R}{R_0}\right)$

$$\begin{aligned} \frac{4}{3}\pi R_0^3 \left(1 + \frac{\Delta R}{R_0}\right)^3 \rho_a g &\geq 4\pi R_0^2 \rho_s t g + \frac{4}{3}\pi R_0^3 \rho_a g \\ \frac{4}{3}\pi (3\Delta R) \rho_a g &\geq 4\pi R_0^2 \rho_s t g \\ \frac{4}{3}\pi \frac{3q^2}{96\pi^2 \varepsilon_0 R_0 P_a} \rho_a g &\geq 4\pi R_0^2 \rho_s t g \\ q^2 &\geq \frac{96\pi^2 R_0^3 \rho_s t \varepsilon_0 P_a}{\rho_a} \end{aligned} \quad \dots (22)$$

$$q \approx 256 \times 10^{-9} \text{ C} \approx 256 \text{ nC}$$

Note that if the surface tension term is retained, we get

$$R_1 \approx \left(1 + \frac{q^2 / 96\pi^2 \varepsilon_0 R_0^4 P_a}{\left[1 + \frac{2}{3} \left(\frac{4\gamma}{R_0 P_a} \right) \right]} \right) R_0$$