Theoretical Question 3: To Commemorate the Centenary of Rutherford's Atomic Nucleus: The scattering of an ion by a neutral atom

Questions	**Points**	**Concepts/Details**
3.1 (Total 1.2)	**0.3**	**3.1a** Use Coulomb's law - Write down inverse square law (0.2 pt) - Correct constant (0.1 pt)
	0.3	**3.1b** Take electric field from 2 charges - Write down superposition of electric field (0.2 pt) - Correct charge polarity/direction (0.1 pt)
	0.3	**3.1c** Correct distances - If the student didn't use the figure provided (-0.1pt)
	0.3	**3.1d Answer**: $E_p = +\frac{4qa}{4\pi\varepsilon_0 r^3}$ or $+\frac{qa}{\pi\varepsilon_0 r^3}$ or $\frac{2p}{4\pi\varepsilon_0 r^3}$
3.2 (Total 3.0)	**0.3**	**3.2a** Write down that the force is the product of electric field and charge. { $\vec{f} = Q\vec{E}_p$ }
	0.4	**3.2b Answer**: $\vec{f} = +\frac{4qa}{4\pi\varepsilon_0 r^3} Q\,\hat{r}$ or $+\frac{qa}{\pi\varepsilon_0 r^3} Q\,\hat{r}$ or $\frac{2p}{4\pi\varepsilon_0 r^3} Q\,\hat{r}$
	0.5	**3.2c** Use the electric field seen by the atom from the ion
	0.4	**3.2d** Use Coulomb's law to write down $\vec{E}_{ion} = -\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ (magnitude 0.1 pt, sign 0.3 pt)
	0.2	**3.2e** Use the given expression for polarisability and write down $\vec{p} = \alpha\vec{E}_{ion} = -\frac{\alpha Q}{4\pi\varepsilon_0 r^2}\hat{r}$
	0.5	**3.2f** Use the concept of induced dipole by substituting $\vec{p} = -\frac{\alpha Q}{4\pi\varepsilon_0 r^2}\hat{r}$ in equation (2) of question (3.1) { $\vec{E}_p = \frac{1}{4\pi\varepsilon_0 r^3}\left[-\frac{2\alpha Q}{4\pi\varepsilon_0 r^2}\hat{r}\right]$ }…….(0.3 pt) Get $\vec{E}_p = -\frac{\alpha Q}{8\pi^2\varepsilon_0^2 r^5}\hat{r}$ (magnitude 0.1 pt, sign 0.1 pt)
	0.3	**3.2g Answer**: $\vec{f} = -\frac{2\alpha Q^2}{(4\pi\varepsilon_0)^2 r^5}\hat{r} = -\frac{\alpha Q^2}{8\pi^2\varepsilon_0^2 r^5}\hat{r}$
	0.2	**3.2h** Point out that the negative sign implies attractive force.

	0.2	**3.2i** Point out Q^2 implies that it is regardless of the sign of the ion.
3.3 (Total 0.9)	**0.5**	**3.3a** Use the definition of potential energy to write down $U = \int_r^\infty \vec{f}.d\vec{r}$ (wrong limit -0.2 pt)
	0.4	**3.3b Answer**: $U = -\frac{\alpha Q^2}{32\pi^2\varepsilon_0^2 r^4}$ (magnitude 0.2 pt, sign 0.2 pt)
3.4 (Total 2.4)	**0.6**	**3.4a** State conservation of angular momentum (0.3 pt) Write down $mv_{\max} r_{\min} = mv_0 b$ (0.3 pt)
	0.6	**3.4b** State conservation of mechanical energy(0.3 pt) Write down $\frac{1}{2}mv_{\max}^2 + \frac{-\alpha Q^2}{32\pi^2\varepsilon_0^2 r^4} = \frac{1}{2}mv_0^2$ (0.3 pt)
	0.5	**3.4c** Substituting $v_{\max}$ in term of $r_{\min}$ (0.1pt) Arrange in term of quadratic equation (0.2 pt) **Answer:** $r_{\min} = \frac{b}{\sqrt{2}}\left[1 \pm \sqrt{1 - \frac{\alpha Q^2}{4\pi^2\varepsilon_0^2 m v_0^2 b^4}}\right]^{\frac{1}{2}}$ (0.2pt)
	0.2	**3.4d** Choose "+" sign and write down $r_{\min} = \frac{b}{\sqrt{2}}\left[1 + \sqrt{1 - \frac{\alpha Q^2}{4\pi^2\varepsilon_0^2 m v_0^2 b^4}}\right]^{\frac{1}{2}}$
	0.5	**3.4e** State the reasoning of the sign with $Q = 0$ or $\alpha \le 0$
3.5 (Total 2.5)	**1.4**	**3.5a** Recognize that a spiral trajectory happens when $r_{\min}$ is imaginary because $b < b_0$. (0.7 pt) Recognize that $r_{\min}$ is imaginary when $1 - \frac{\alpha Q^2}{4\pi^2\varepsilon_0^2 m v_0^2 b^4} < 0$ (0.7 pt)
	0.7	**3.5b** Write down $b < b_0 = \left(\frac{\alpha Q^2}{4\pi^2\varepsilon_0^2 m v_0^2}\right)^{\frac{1}{4}}$
	0.4	**3.5c Answer**: $A = \pi\left(\frac{\alpha Q^2}{4\pi^2\varepsilon_0^2 m v_0^2}\right)^{\frac{1}{2}}$