SOLUTIONS to Theory Question 1

Geometry Each side of the diamond has length $L = \dfrac{a}{\cos\theta}$ and the distance between parallel sides is $D = \dfrac{a}{\cos\theta}\sin(2\theta) = 2a\sin\theta$. The area is the product thereof, $A = LD$, giving

1.1

$$A = 2a^2\tan\theta\,.$$

The height H by which a tilt of ϕ lifts OUT1 above IN is $H = D\sin\phi$ or

1.2

$$H = 2a\sin\theta\,\sin\phi\,.$$

Optical path length Only the two parallel lines for IN and OUT1 matter, each having length L. With the de Broglie wavelength λ_0 on the IN side and λ_1 on the OUT1 side, we have

$$\Delta N_{\text{opt}} = \frac{L}{\lambda_0} - \frac{L}{\lambda_1} = \frac{a}{\lambda_0\cos\theta}\left(1 - \frac{\lambda_0}{\lambda_1}\right).$$

The momentum is h/λ_0 or h/λ_1, respectively, and the statement of energy conservation reads

$$\frac{1}{2M}\left(\frac{h}{\lambda_0}\right)^2 = \frac{1}{2M}\left(\frac{h}{\lambda_1}\right)^2 + MgH\,,$$

which implies

$$\frac{\lambda_0}{\lambda_1} = \sqrt{1 - 2\frac{gM^2}{h^2}\lambda_0^2 H}\,.$$

Upon recognizing that $(gM^2/h^2)\lambda_0^2 H$ is of the order of 10^{-7}, this simplifies to

$$\frac{\lambda_0}{\lambda_1} = 1 - \frac{gM^2}{h^2}\lambda_0^2 H\,,$$

and we get

$$\Delta N_{\text{opt}} = \frac{a}{\lambda_0\cos\theta}\frac{gM^2}{h^2}\lambda_0^2 H$$

or

1.3

$$\Delta N_{\text{opt}} = 2\frac{gM^2}{h^2}a^2\lambda_0 \tan\theta \, \sin\phi \, .$$

A more compact way of writing this is

1.4

$$\Delta N_{\text{opt}} = \frac{\lambda_0 A}{V}\sin\phi \, ,$$

where

1.4

$$V = 0.1597 \times 10^{-13}\,\text{m}^3 = 0.1597\,\text{nm}\,\text{cm}^2$$

is the numerical value for the volume parameter V.

There is constructive interference (high intensity in OUT1) when the optical path lengths of the two paths differ by an integer, $\Delta N_{\text{opt}} = 0, \pm 1, \pm 2, \ldots$, and we have destructive interference (low intensity in OUT1) when they differ by an integer plus half, $\Delta N_{\text{opt}} = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$. Changing ϕ from $\phi = -90°$ to $\phi = 90°$ gives

$$\Delta N_{\text{opt}}\Big|_{\phi=-90°}^{\phi=90°} = \frac{2\lambda_0 A}{V} \, ,$$

which tell us that

1.5

$$\sharp \text{ of cycles} = \frac{2\lambda_0 A}{V} \, .$$

Experimental data For $a = 3.6\,\text{cm}$ and $\theta = 22.1°$ we have $A = 10.53\,\text{cm}^2$, so that

1.6

$$\lambda_0 = \frac{19 \times 0.1597}{2 \times 10.53}\,\text{nm} = 0.1441\,\text{nm} \, .$$

And 30 full cycles for $\lambda_0 = 0.2\,\text{nm}$ correspond to an area

1.7

$$A = \frac{30 \times 0.1597}{2 \times 0.2}\,\text{cm}^2 = 11.98\,\text{cm}^2 \, .$$