PLANCK’S CONSTANT IN THE LIGHT OF AN INCANDESCENT LAMP
SOLUTION

TASK 1

Draw the electric connections in the boxes and between boxes below.
**TASK 2**

a)

<table>
<thead>
<tr>
<th>$t_0 = 24 , ^\circ C$</th>
<th>$T_0 = 297 , K$</th>
<th>$\Delta T_0 = 1 , K$</th>
</tr>
</thead>
</table>

b)

<table>
<thead>
<tr>
<th>$V / \text{mV}$</th>
<th>$I / \text{mA}$</th>
<th>$R_B / \Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.9</td>
<td>1.87</td>
<td>11.7</td>
</tr>
<tr>
<td>30.5</td>
<td>2.58</td>
<td>11.8</td>
</tr>
<tr>
<td>34.9</td>
<td>2.95</td>
<td>11.8</td>
</tr>
<tr>
<td>37.0</td>
<td>3.12</td>
<td>11.9</td>
</tr>
<tr>
<td>40.1</td>
<td>3.37</td>
<td>11.9</td>
</tr>
<tr>
<td>43.0</td>
<td>3.60</td>
<td>11.9</td>
</tr>
<tr>
<td>47.6</td>
<td>3.97</td>
<td>12.0</td>
</tr>
<tr>
<td>51.1</td>
<td>4.24</td>
<td>12.1</td>
</tr>
<tr>
<td>55.3</td>
<td>4.56</td>
<td>12.1</td>
</tr>
<tr>
<td>58.3</td>
<td>4.79</td>
<td>12.2</td>
</tr>
<tr>
<td>61.3</td>
<td>5.02</td>
<td>12.2</td>
</tr>
<tr>
<td>65.5</td>
<td>5.33</td>
<td>12.3</td>
</tr>
<tr>
<td>67.5</td>
<td>5.47</td>
<td>12.3</td>
</tr>
<tr>
<td>73.0</td>
<td>5.88</td>
<td>12.4</td>
</tr>
<tr>
<td>80.9</td>
<td>6.42</td>
<td>12.6</td>
</tr>
<tr>
<td>85.6</td>
<td>6.73</td>
<td>12.7</td>
</tr>
<tr>
<td>89.0</td>
<td>6.96</td>
<td>12.8</td>
</tr>
<tr>
<td>95.1</td>
<td>7.36</td>
<td>12.9</td>
</tr>
<tr>
<td>111.9</td>
<td>8.38</td>
<td>13.4</td>
</tr>
<tr>
<td>130.2</td>
<td>9.37</td>
<td>13.9</td>
</tr>
<tr>
<td>181.8</td>
<td>11.67</td>
<td>15.6</td>
</tr>
<tr>
<td>220</td>
<td>13.04</td>
<td>16.9</td>
</tr>
<tr>
<td>307</td>
<td>15.29</td>
<td>20.1</td>
</tr>
<tr>
<td>447</td>
<td>17.68</td>
<td>25.1</td>
</tr>
<tr>
<td>590</td>
<td>19.8</td>
<td>29.8</td>
</tr>
<tr>
<td>730</td>
<td>21.5</td>
<td>33.9</td>
</tr>
<tr>
<td>860</td>
<td>23.2</td>
<td>37.1</td>
</tr>
<tr>
<td>960</td>
<td>24.4</td>
<td>39.3</td>
</tr>
</tbody>
</table>

$V_{\text{min}} = 9.2 \, \text{mV}$

* This is a characteristic of your apparatus. You can’t go below it.

We represent $R_B$ in the vertical axis against $I$. 
In order to work out $R_{B0}$, we choose the first ten readings.

**TASK 2**

c)

<table>
<thead>
<tr>
<th>$V$ / mV</th>
<th>$I$ / mA</th>
<th>$R_B$ / Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.9 ± 0.1</td>
<td>1.87 ± 0.01</td>
<td>11.7 ± 0.1</td>
</tr>
<tr>
<td>30.5 ± 0.1</td>
<td>2.58 ± 0.01</td>
<td>11.8 ± 0.1</td>
</tr>
<tr>
<td>34.9 ± 0.1</td>
<td>2.95 ± 0.01</td>
<td>11.8 ± 0.1</td>
</tr>
<tr>
<td>37.0 ± 0.1</td>
<td>3.12 ± 0.01</td>
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<tr>
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<td>3.37 ± 0.01</td>
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</tr>
<tr>
<td>43.0 ± 0.1</td>
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</tr>
<tr>
<td>47.6 ± 0.1</td>
<td>3.97 ± 0.01</td>
<td>12.0 ± 0.1</td>
</tr>
<tr>
<td>51.1 ± 0.1</td>
<td>4.24 ± 0.01</td>
<td>12.1 ± 0.1</td>
</tr>
<tr>
<td>55.3 ± 0.1</td>
<td>4.56 ± 0.01</td>
<td>12.1 ± 0.1</td>
</tr>
<tr>
<td>58.3 ± 0.1</td>
<td>4.79 ± 0.01</td>
<td>12.2 ± 0.1</td>
</tr>
</tbody>
</table>
Error for $R_B$ (We work out the error for first value, as example).

$$\Delta R_B = R_B \sqrt{\frac{(\Delta V)}{V}^2 + \left(\frac{\Delta I}{I}\right)^2} = 11.71 \sqrt{\left(\frac{0.1}{21.9}\right)^2 + \left(\frac{0.01}{1.87}\right)^2} = 0.1$$

We have worked out $R_{B0}$ by the least squares.

$R_{B0} = 11.4$

slope $m = 0.167$

$\sum I^2 = 130.38$

$\sum I = 35.05$

$n = 10$

For axis $X$ : $\sigma_I = \sqrt{\frac{\sum \Delta I^2}{n}} = 0.01$

For axis $Y$ : $\sigma_{R_B} = \sqrt{\frac{\sum \Delta R_B^2}{n}} = 0.047$

$\sigma = \sqrt{\sigma_{R_B}^2 + m^2 \sigma_I^2} = \sqrt{0.1^2 + 0.167^2 \cdot 0.01^2} = 0.1$

$$\Delta R_{B0} = \frac{\sigma^2 \sum I^2}{n \sum I^2 - (\sum I)^2} = \sqrt{\frac{0.1^2 \times 130.38}{10 \cdot 130.38 - 35.05^2}} = 0.13$$

$R_{B0} = 11.4 \Omega \quad \Delta R_{B0} = 0.1 \Omega$
d) \( T = aR^{0.83} \); \( a = \frac{T_0}{R_0^{0.83}} \); \( a = \frac{297}{11.4^{0.83}} = 39.40 \)

Working out the error for two methods:

**Method A**

\[
\ln a = \ln T_0 - 0.83 \ln R_{B0}; \quad \Delta a = a\left(\frac{\Delta T_0}{T_0} + 0.83 \frac{\Delta R_{B0}}{R_{B0}}\right); \quad \Delta a = 39.40\left(\frac{1}{297} + 0.83 \frac{0.1}{11.4}\right) = 0.419 = 0.4
\]

**Method B**

Higher value of \( a \): \( a_{\text{max}} = \frac{T_0 + \Delta T_0}{(R_0 - \Delta R_0)^{0.83}} = \frac{297 + 1}{(11.4 - 0.1)^{0.83}} = 39.8255 \)

Smaller value of \( a \): \( a_{\text{min}} = \frac{T_0 - \Delta T_0}{(R_0 + \Delta R_0)^{0.83}} = \frac{297 - 1}{(11.4 + 0.1)^{0.83}} = 38.9863 \)

\[
\Delta a = \frac{a_{\text{max}} - a_{\text{min}}}{2} = \frac{39.8255 - 38.9863}{2} = 0.419 = 0.4
\]

\[
a = 39.4 \quad \Delta a = 0.4
\]

**TASK 3**

Because of \( 2\Delta \lambda = 620 - 565 \); \( \Delta \lambda = 28 \) nm

\[
\lambda_0 = 590 \text{ nm} \quad \Delta \lambda = 28 \text{ nm}
\]

**TASK 4**

a)

<table>
<thead>
<tr>
<th>( V ) / V</th>
<th>( I ) / mA</th>
<th>( R ) / k(\Omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.48</td>
<td>85.5</td>
<td>8.77</td>
</tr>
<tr>
<td>9.73</td>
<td>86.8</td>
<td>8.11</td>
</tr>
<tr>
<td>9.83</td>
<td>87.3</td>
<td>7.90</td>
</tr>
<tr>
<td>10.01</td>
<td>88.2</td>
<td>7.49</td>
</tr>
<tr>
<td>10.25</td>
<td>89.4</td>
<td>7.00</td>
</tr>
<tr>
<td>10.41</td>
<td>90.2</td>
<td>6.67</td>
</tr>
<tr>
<td>10.61</td>
<td>91.2</td>
<td>6.35</td>
</tr>
<tr>
<td>10.72</td>
<td>91.8</td>
<td>6.16</td>
</tr>
<tr>
<td>10.82</td>
<td>92.2</td>
<td>6.01</td>
</tr>
<tr>
<td>10.97</td>
<td>93.0</td>
<td>5.77</td>
</tr>
<tr>
<td>11.03</td>
<td>93.3</td>
<td>5.69</td>
</tr>
<tr>
<td>11.27</td>
<td>94.5</td>
<td>5.35</td>
</tr>
<tr>
<td>11.42</td>
<td>95.1</td>
<td>5.17</td>
</tr>
<tr>
<td>11.50</td>
<td>95.5</td>
<td>5.07</td>
</tr>
</tbody>
</table>
b)  

Because of \( \ln \frac{R}{R'} = \gamma \ln 0.512 \); \( \gamma = \ln \frac{R}{R'}/\ln 0.512 = \frac{5.07}{8.11}/\ln 0.512 = 0.702 \)

For working out \( \Delta \gamma \) we know that:
\[
R \pm \Delta R = 5.07 \pm 0.01 \text{ k}\Omega \\
R' \pm \Delta R' = 8.11 \pm 0.01 \text{ k}\Omega \\
\text{Transmittance, } t = 51.2 \% 
\]

Working out the error for two methods:

**Method A**

\[
\gamma = \frac{\ln R/R'}{\ln t} ; \quad \Delta \gamma = \frac{1}{\ln t} \left( \frac{\Delta R + \Delta R'}{R} + \frac{\Delta R + \Delta R'}{R'} \right) \approx \frac{1}{\ln 0.512} \left( \frac{0.01}{5.07} + \frac{0.01}{8.11} \right) = 0.00479 ; \quad \Delta \gamma = 0.005
\]

**Method B**

Higher value of \( \gamma \):
\[
\gamma_{\text{max}} = \frac{\ln \frac{R - \Delta R}{R' + \Delta R'}}{\ln t} \approx \ln \frac{5.07 - 0.01}{8.11 + 0.01}/\ln 0.512 = 0.70654 
\]

Smaller value of \( \gamma \):
\[
\gamma_{\text{min}} = \frac{\ln \frac{R + \Delta R}{R' - \Delta R'}}{\ln t} \approx \ln \frac{5.07 + 0.01}{8.11 - 0.01}/\ln 0.512 = 0.69696 
\]

\[
\Delta \gamma = \frac{\gamma_{\text{max}} - \gamma_{\text{min}}}{2} = \frac{0.70654 - 0.69696}{2} = 0.00479 ; \quad \Delta \gamma = 0.005
\]

\[
\begin{array}{|c|c|}
\hline
R &= 5.07 \text{ k}\Omega \\
R' &= 8.11 \text{ k}\Omega \\
\gamma &= 0.702 \\
\Delta \gamma &= 0.005 \\
\hline
\end{array}
\]

c)  

We know that \( R = c_3 e^{\frac{c_2 \gamma}{\lambda_0 T}} \) (3)

then \( \ln R = \ln c_3 + \frac{c_2 \gamma}{\lambda_0 T} \)

Because of \( T = aR_B^{0.83} \) (6)

consequently \( \ln R = \ln c_3 + \frac{c_2 \gamma}{\lambda_0 a} R_B^{-0.83} \)

\[
\ln R = \ln c_3 + \frac{c_2 \gamma}{\lambda_0 a} R_B^{-0.83} \quad \text{Eq. (9)}
\]
d)  

<table>
<thead>
<tr>
<th>$V/V$</th>
<th>$I/mA$</th>
<th>$R_B/\Omega$</th>
<th>$T/K$</th>
<th>$R_B^{0.83}$ (S.I.)</th>
<th>$R/k\Omega$</th>
<th>$\ln R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.48 ± 0.01</td>
<td>85.5 ± 0.1</td>
<td>110.9 ± 0.2</td>
<td>1962 ± 18</td>
<td>$(2.008 ± 0.004) \times 10^2$</td>
<td>8.77 ± 0.01</td>
<td>2.171 ± 0.001</td>
</tr>
<tr>
<td>9.73 ± 0.01</td>
<td>86.8 ± 0.1</td>
<td>112.1 ± 0.2</td>
<td>1980 ± 18</td>
<td>$(1.990±0.004)\times10^2$</td>
<td>8.11 ± 0.01</td>
<td>2.093 ± 0.001</td>
</tr>
<tr>
<td>9.83 ± 0.01</td>
<td>87.3 ± 0.1</td>
<td>112.6 ± 0.2</td>
<td>1987 ± 18</td>
<td>$(1.983±0.004)\times10^2$</td>
<td>7.90 ± 0.01</td>
<td>2.067 ± 0.001</td>
</tr>
<tr>
<td>10.01 ± 0.01</td>
<td>88.2 ± 0.1</td>
<td>113.5 ± 0.2</td>
<td>2000 ± 18</td>
<td>$(1.970±0.004)\times10^2$</td>
<td>7.49 ± 0.01</td>
<td>2.014 ± 0.001</td>
</tr>
<tr>
<td>10.25 ± 0.01</td>
<td>89.4 ± 0.1</td>
<td>114.7 ± 0.2</td>
<td>2018 ± 18</td>
<td>$(1.952±0.003)\times10^2$</td>
<td>7.00 ± 0.01</td>
<td>1.946 ± 0.001</td>
</tr>
<tr>
<td>10.41 ± 0.01</td>
<td>90.2 ± 0.1</td>
<td>115.4 ± 0.2</td>
<td>2028 ± 18</td>
<td>$(1.943±0.003)\times10^2$</td>
<td>6.67 ± 0.01</td>
<td>1.894 ± 0.002</td>
</tr>
<tr>
<td>10.61 ± 0.01</td>
<td>91.2 ± 0.1</td>
<td>116.3 ± 0.2</td>
<td>2041 ± 18</td>
<td>$(1.930±0.003)\times10^2$</td>
<td>6.35 ± 0.01</td>
<td>1.849 ± 0.002</td>
</tr>
<tr>
<td>10.72 ± 0.01</td>
<td>91.8 ± 0.1</td>
<td>116.8 ± 0.2</td>
<td>2049 ± 19</td>
<td>$(1.923±0.003)\times10^2$</td>
<td>6.16 ± 0.01</td>
<td>1.818 ± 0.002</td>
</tr>
<tr>
<td>10.82 ± 0.01</td>
<td>92.2 ± 0.1</td>
<td>117.4 ± 0.2</td>
<td>2057 ± 19</td>
<td>$(1.915±0.003)\times10^2$</td>
<td>6.01 ± 0.01</td>
<td>1.793 ± 0.002</td>
</tr>
<tr>
<td>10.97 ± 0.01</td>
<td>93.0 ± 0.1</td>
<td>118.0 ± 0.2</td>
<td>2066 ± 19</td>
<td>$(1.907±0.003)\times10^2$</td>
<td>5.77 ± 0.01</td>
<td>1.753 ± 0.002</td>
</tr>
<tr>
<td>11.03 ± 0.01</td>
<td>93.3 ± 0.1</td>
<td>118.2 ± 0.2</td>
<td>2069 ± 19</td>
<td>$(1.904±0.003)\times10^2$</td>
<td>5.69 ± 0.01</td>
<td>1.739 ± 0.002</td>
</tr>
<tr>
<td>11.27 ± 0.01</td>
<td>94.5 ± 0.1</td>
<td>119.3 ± 0.2</td>
<td>2085 ± 19</td>
<td>$(1.890±0.003)\times10^2$</td>
<td>5.35 ± 0.01</td>
<td>1.677 ± 0.002</td>
</tr>
<tr>
<td>11.42 ± 0.01</td>
<td>95.1 ± 0.1</td>
<td>120.1 ± 0.2</td>
<td>2096 ± 19</td>
<td>$(1.880±0.003)\times10^2$</td>
<td>5.15 ± 0.01</td>
<td>1.639 ± 0.002</td>
</tr>
<tr>
<td>11.50 ± 0.01</td>
<td>95.5 ± 0.1</td>
<td>120.4 ± 0.2</td>
<td>2101 ± 19</td>
<td>$(1.875±0.003)\times10^2$</td>
<td>5.07 ± 0.01</td>
<td>1.623 ± 0.002</td>
</tr>
</tbody>
</table>

We work out the errors for all the first row, as example.

Error for $R_B$:  
$$\Delta R_B = R_B \sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta I}{I}\right)^2} = 110.9 \sqrt{\left(\frac{0.01}{9.48}\right)^2 + \left(\frac{0.1}{85.5}\right)^2} = 0.2 \ \Omega$$

Error for $T$:  
$$\Delta T = T \left(\frac{\Delta a}{a} + 0.83 \frac{\Delta R_B}{R_B}\right); \ \Delta T = 1962 \left(\frac{0.3}{39.4} + 0.83 \frac{0.2}{110.9}\right) = 18 K$$

Error for $R_B^{0.83}$:  
$$x = R_B^{0.83}; \ \ln x = -0.83 \ln R_B; \ \Delta x = x \cdot 0.83 \frac{\Delta R_B}{R_B} = \left(R_B^{0.83}\right) = R_B^{0.83} \frac{\Delta R_B}{R_B}$$

$$\Delta \left(R_B^{0.83}\right) = 0.020077 \cdot \frac{0.2}{110.9} = 0.004 \times 10^{-2}$$

Error for $\ln R$:  
$$\Delta \ln R = \frac{\Delta R}{R}, \ \Delta \ln R = \frac{0.01}{8.77} = 0.001$$

e)  

We plot $\ln R$ versus $R_B^{0.83}$.
By the least squares
Slope = $m = 414.6717$

\[
\sum (R_B^{-0.83})^2 = 5.23559 \times 10^{-3}
\]
\[
\sum R_B^{-0.83} = 0.27068
\]

$n = 14$

For axis $X$: $\sigma_{R_B^{-0.83}} = \sqrt{\frac{\sum \Delta (R_B^{-0.83})^2}{n}} = 0.003 \times 10^{-2}$

For axis $Y$: $\sigma_{\ln R} = \sqrt{\frac{\sum \Delta (\ln R)^2}{n}} = 0.002$

\[
\sigma = \sqrt{\sigma_{\ln R}^2 + m^2 \sigma_{R_B^{-0.83}}^2} = \sqrt{0.002^2 + 414.672^2 \cdot (0.003 \times 10^{-2})^2} = 0.0126
\]

\[
\Delta m = \sqrt{n \sum (R_B^{-0.83})^2 - (\sum R_B^{-0.83})^2} = \sqrt{\frac{14 \cdot 0.0126^2}{14 \cdot 5.23559 \times 10^{-3} - (0.27068)^2}} = 8.295
\]

Because of

\[
m = \frac{c_2 \gamma}{\lambda_0 \alpha}
\]

and

\[
c_2 = \frac{hc}{k}
\]

then

\[
h = \frac{mk \lambda_0 \alpha}{c \gamma}
\]
\[ h = \frac{414.67 \cdot 1.381 \times 10^{-23} \cdot 590 \times 10^{-9} \cdot 39.4}{2.998 \times 10^8 \cdot 0.702} = 6.33 \times 10^{-34} \]

\[ \Delta h = h \left( \frac{\Delta m}{m} \right)^2 + \left( \frac{\Delta k}{k} \right)^2 + \left( \frac{\Delta \lambda_0}{\lambda_0} \right)^2 + \left( \frac{\Delta \alpha}{\alpha} \right)^2 + \left( \frac{\Delta \gamma}{\gamma} \right)^2 \]

\[ \Delta h = 6.34 \times 10^{-34} \left( \frac{8.3}{415} \right)^2 + 0 + \left( \frac{28}{590} \right)^2 + \left( \frac{0.3}{39.4} \right)^2 + 0 + \left( \frac{0.01}{0.70} \right)^2 = 0.34 \times 10^{-34} \]

\[ h = 6.3 \times 10^{-34} \text{ J} \cdot \text{s} \quad \Delta h = 0.3 \times 10^{-34} \text{ J} \cdot \text{s} \]