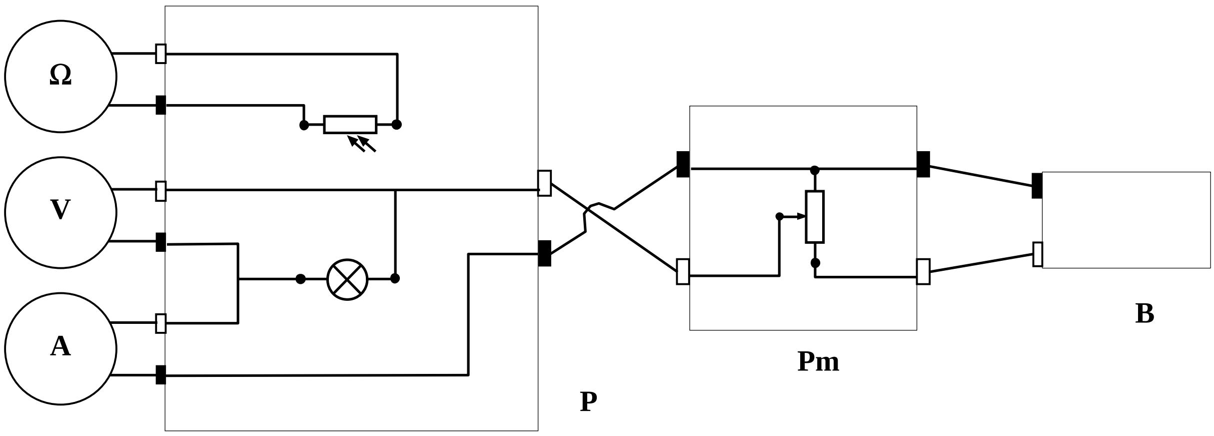
# PLANCK'S CONSTANT IN THE LIGHT OF AN INCANDESCENT LAMP
## SOLUTION

### TASK 1

Draw the electric connections in the boxes and between boxes below.




| | |
|---|---|
| **Photoresistor** | |
| **Incandescent Bulb** | |
| **Potentiometer** | |
| **Red socket** | ▯ |
| **Black socket** | ▮ |

| | |
|---|---|
| **Ω** | **Ohmmeter** |
| **V** | **Voltmeter** |
| **A** | **Ammeter** |
| **P** | **Platform** |
| **Pm** | **Potentiometer** |
| **B** | **Battery** |

*TASK 2*

a)

| $t_0$ = 24 ºC | $T_0$ = 297 K | $\Delta T_0$ = 1 K |
|---|---|---|

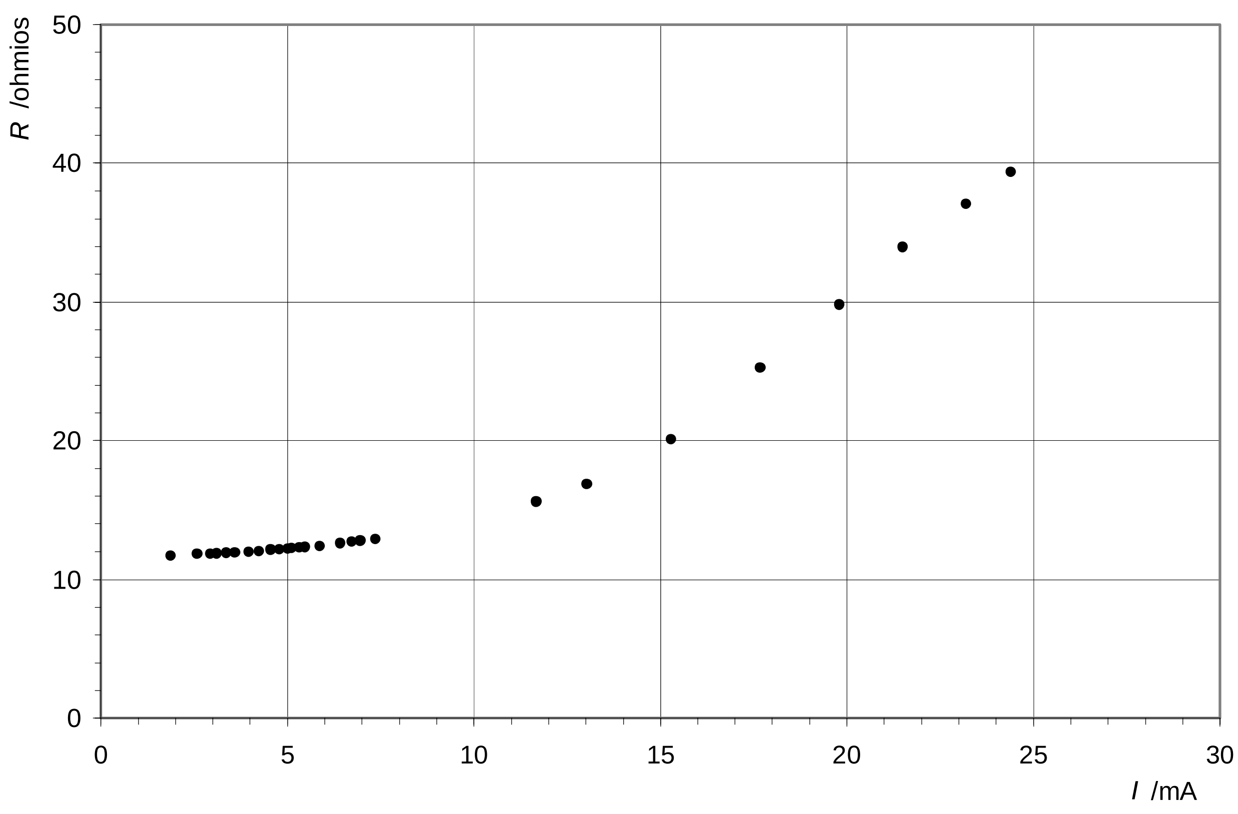
b)

| $V$ /mV | $I$ / mA | $R_B$ /Ω |
|---|---|---|
| 21.9 | 1.87 | 11.7 |
| 30.5 | 2.58 | 11.8 |
| 34.9 | 2.95 | 11.8 |
| 37.0 | 3.12 | 11.9 |
| 40.1 | 3.37 | 11.9 |
| 43.0 | 3.60 | 11.9 |
| 47.6 | 3.97 | 12.0 |
| 51.1 | 4.24 | 12.1 |
| 55.3 | 4.56 | 12.1 |
| 58.3 | 4.79 | 12.2 |
| 61.3 | 5.02 | 12.2 |
| 65.5 | 5.33 | 12.3 |
| 67.5 | 5.47 | 12.3 |
| 73.0 | 5.88 | 12.4 |
| 80.9 | 6.42 | 12.6 |
| 85.6 | 6.73 | 12.7 |
| 89.0 | 6.96 | 12.8 |
| 95.1 | 7.36 | 12.9 |
| 111.9 | 8.38 | 13.4 |
| 130.2 | 9.37 | 13.9 |
| 181.8 | 11.67 | 15.6 |
| 220 | 13.04 | 16.9 |
| 307 | 15.29 | 20.1 |
| 447 | 17.68 | 25.1 |
| 590 | 19.8 | 29.8 |
| 730 | 21.5 | 33.9 |
| 860 | 23.2 | 37.1 |
| 960 | 24.4 | 39.3 |

$V_{min}$ = 9.2 mV *

* This is a characteristic of your apparatus. You can´t go below it.

We represent $R_B$ in the vertical axis against $I$.

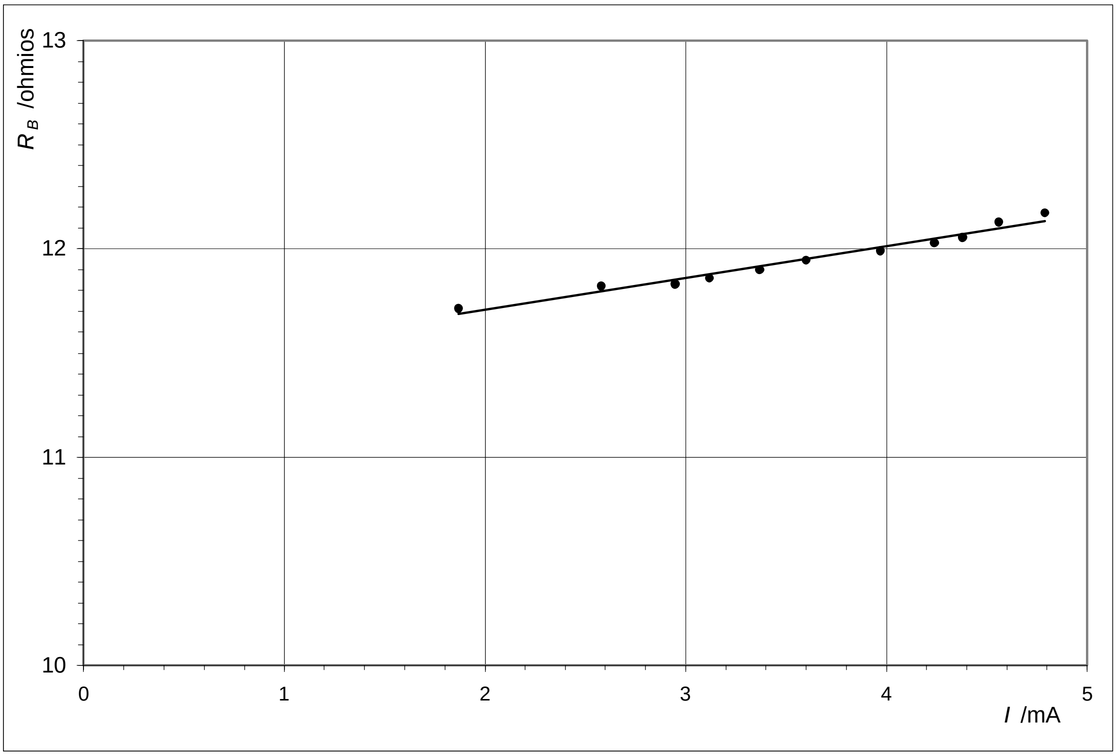



In order to work out $R_{B0}$ , we choose the first ten readings.

*TASK 2*

c)

| $V$ /mV | $I$ / mA | $R_B$ /Ω |
|---|---|---|
| 21.9 ± 0.1 | 1.87 ± 0.01 | 11.7 ± 0.1 |
| 30.5 ± 0.1 | 2.58 ± 0.01 | 11.8 ± 0.1 |
| 34.9 ± 0.1 | 2.95 ± 0.01 | 11.8 ± 0.1 |
| 37.0 ± 0.1 | 3.12 ± 0.01 | 11.9 ± 0.1 |
| 40.1 ± 0.1 | 3.37 ± 0.01 | 11.9 ± 0.1 |
| 43.0 ± 0.1 | 3.60 ± 0.01 | 11.9 ± 0.1 |
| 47.6 ± 0.1 | 3.97 ± 0.01 | 12.0 ± 0.1 |
| 51.1 ± 0.1 | 4.24 ± 0.01 | 12.1 ± 0.1 |
| 55.3 ± 0.1 | 4.56 ± 0.01 | 12.1 ± 0.1 |
| 58.3 ± 0.1 | 4.79 ± 0.01 | 12.2 ± 0.1 |




Error for $R_B$ (We work out the error for first value, as example).

$$\Delta R_B = R_B\sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta I}{I}\right)^2} = 11.71\sqrt{\left(\frac{0.1}{21.9}\right)^2 + \left(\frac{0.01}{1.87}\right)^2} = 0.1$$

We have worked out $R_{B0}$ by the least squares.

$$R_{B0} = 11.4$$
$$\text{slope} = m = 0.167$$
$$\sum I^2 = 130.38$$
$$\sum I = 35.05$$
$$n = 10$$

$$\text{For axis } X : \sigma_I = \sqrt{\frac{\sum \Delta I^2}{n}} = 0.01$$

$$\text{For axis } Y : \sigma_{R_B} = \sqrt{\frac{\sum \Delta R_B^2}{n}} = 0.047$$

$$\sigma = \sqrt{\sigma_{R_B}^2 + m^2\sigma_I^2} = \sqrt{0.1^2 + 0.167^2 \cdot 0.01^2} = 0.1$$

$$\Delta R_{B0} = \sqrt{\frac{\sigma^2 \sum I^2}{n\sum I^2 - \left(\sum I\right)^2}} = \sqrt{\frac{0.1^2 \times 130.38}{10 \cdot 130.38 - 35.05^2}} = 0.13$$

| | |
|---|---|
| $R_{B0}$ = 11,4 Ω | $\Delta R_{B0}$ = 0.1 Ω |

d) $T = aR^{0.83}$ ; $a = \frac{T_0}{R_0^{0.83}}$; $a = \frac{297}{11.4^{0.83}} = 39.40$

Working out the error for two methods:

Method A

$\ln a = \ln T_0 - 0.83 \ln R_{B0}$ ; $\Delta a = a\left(\frac{\Delta T_0}{T_0} + 0.83\frac{\Delta R_{B0}}{R_{B0}}\right)$ ; $\Delta a = 39.40\left(\frac{1}{297} + 0.83\frac{0.1}{11.40}\right) = 0.419 = 0.4$

Method B

Higher value of $a$: $a_{max} = \frac{T_0 + \Delta T_0}{(R_0 - \Delta R_0)^{0.83}} = \frac{297+1}{(11.4-0.1)^{0.83}} = 39.8255$

Smaller value of $a$: $a_{min} = \frac{T_0 - \Delta T_0}{(R_0 + \Delta R_0)^{0.83}} = \frac{297-1}{(11.4+0.1)^{0.83}} = 38.9863$

$$\Delta a = \frac{a_{max} - a_{min}}{2} = \frac{39.8255 - 38.9863}{2} = 0.419 = 0.4$$

| | |
|---|---|
| $a$ = 39.4 | $\Delta a$ = 0.4 |

*TASK 3*

Because of $2\Delta\lambda = 620 - 565$ ; $\Delta\lambda = 28$ nm

| | |
|---|---|
| $\lambda_0$ = 590 nm | $\Delta\lambda$ = 28 nm |

*TASK 4*

a)

| $V$ /V | $I$ / mA | $R$ /kΩ |
|---|---|---|
| 9.48 | 85.5 | 8.77 |
| 9.73 | 86.8 | 8.11 |
| 9.83 | 87.3 | 7.90 |
| 100.1 | 88.2 | 7.49 |
| 10.25 | 89.4 | 7.00 |
| 10.41 | 90.2 | 6.67 |
| 10.61 | 91.2 | 6.35 |
| 10.72 | 91.8 | 6.16 |
| 10.82 | 92.2 | 6.01 |
| 10.97 | 93.0 | 5.77 |
| 11.03 | 93.3 | 5.69 |
| 11.27 | 94.5 | 5.35 |
| 11.42 | 95.1 | 5.17 |
| 11.50 | 95.5 | 5.07 |

b)

Because of $\ln \frac{R}{R'} = \gamma \ln 0.512$ ; $\gamma = \ln \frac{R}{R'} \Big/ \ln 0.512 = \ln \frac{5.07}{8.11} \Big/ \ln 0.512 = 0.702$

For working out $\Delta\gamma$ we know that:

$R \pm \Delta R$ = 5.07 ± 0.01 kΩ

$R' \pm \Delta R'$ = 8.11 ± 0.01 kΩ

Transmittance, $t$ = 51.2 %

Working out the error for two methods:

<u>Method A</u>

$$\gamma = \frac{\ln R/R'}{\ln t} \; ; \; \Delta\gamma = \frac{1}{\ln t}\left(\frac{\Delta R}{R} + \frac{\Delta R'}{R'}\right) = \frac{1}{\ln 0.512}\left(\frac{0.01}{5.07} + \frac{0.01}{8.11}\right) = 0.00479 \; ; \; \Delta\gamma = 0.005$$

<u>Method B</u>

Higher value of $\gamma$: $\gamma_{\max} = \ln \frac{R - \Delta R}{R' + \Delta R'} \Big/ \ln \gamma = \ln \frac{5.07 - 0.01}{8.11 + 0.01} \Big/ \ln 0.512 = 0.70654$

Smaller value of $\gamma$: $\gamma_{\max} = \ln \frac{R + \Delta R}{R' - \Delta R'} \Big/ \ln \gamma = \ln \frac{5.07 + 0.01}{8.11 - 0.01} \Big/ \ln 0.512 = 0.69696$

$$\Delta\gamma = \frac{\gamma_{\max} - \gamma_{\min}}{2} = \frac{0.70654 - 0.69696}{2} = 0.00479; \quad \Delta\gamma = 0.005$$

| | |
|---|---|
| $R$ = 5.07 kΩ | $\gamma$ = 0.702 |
| $R'$ = 8.11 kΩ | $\Delta\gamma$ = 0.005 |

c)

We know that $R = c_3 e^{\frac{c_2 \gamma}{\lambda_0 T}}$ (3)

then $\ln R = \ln c_3 + \frac{c_2 \gamma}{\lambda_0 T}$

Because of $T = a R_B^{0.83}$ (6)

consequently $\ln R = \ln c_3 + \frac{c_2 \gamma}{\lambda_0 a} R_B^{-0.83}$

$$\ln R = \ln c_3 + \frac{c_2 \gamma}{\lambda_0 a} R_B^{-0.83} \quad \text{Eq. (9)}$$

d)

| $V$ /V | $I$ / mA | $R_B$ / Ω | $T$ / K | $R_B^{-0.83}$ (S.I.) | $R$ / kΩ | ln $R$ |
|---|---|---|---|---|---|---|
| 9.48 ± 0.01 | 85.5 ± 0.1 | 110.9 ± 0.2 | 1962 ± 18 | (2.008 ± 0.004)$10^{-2}$ | 8.77 ± 0.01 | 2.171 ± 0.001 |
| 9.73± 0.01 | 86.8 ± 0.1 | 112.1 ± 0.2 | 1980 ± 18 | (1.990± 0.004)$10^{-2}$ | 8.11 ± 0.01 | 2.093 ± 0.001 |
| 9.83± 0.01 | 87.3 ± 0.1 | 112.6 ± 0.2 | 1987 ± 18 | (1.983± 0.004)$10^{-2}$ | 7.90 ± 0.01 | 2.067 ± 0.001 |
| 10.01± 0.01 | 88.2 ± 0.1 | 113.5 ± 0.2 | 2000 ± 18 | (1.970± 0.004)$10^{-2}$ | 7.49 ± 0.01 | 2.014 ± 0.001 |
| 10.25± 0.01 | 89.4 ± 0.1 | 114.7 ± 0.2 | 2018 ± 18 | (1.952± 0.003)$10^{-2}$ | 7.00 ± 0.01 | 1.946 ± 0.001 |
| 10.41± 0.01 | 90.2 ± 0.1 | 115.4 ± 0.2 | 2028 ± 18 | (1.943± 0.003)$10^{-2}$ | 6.67 ± 0.01 | 1.894 ± 0.002 |
| 10.61± 0.01 | 91.2 ± 0.1 | 116.3 ± 0.2 | 2041 ± 18 | (1.930± 0.003)$10^{-2}$ | 6.35 ± 0.01 | 1.849 ± 0.002 |
| 10.72± 0.01 | 91.8 ± 0.1 | 116.8 ± 0.2 | 2049 ± 19 | (1.923± 0.003)$10^{-2}$ | 6.16 ± 0.01 | 1.818 ± 0.002 |
| 10.82± 0.01 | 92.2 ± 0.1 | 117.4 ± 0.2 | 2057 ± 19 | (1.915± 0.003)$10^{-2}$ | 6.01 ± 0.01 | 1.793 ± 0.002 |
| 10.97± 0.01 | 93.0 ± 0.1 | 118.0 ± 0.2 | 2066 ± 19 | (1.907± 0.003)$10^{-2}$ | 5.77 ± 0.01 | 1.753 ± 0.002 |
| 11.03± 0.01 | 93.3 ± 0.1 | 118.2 ± 0.2 | 2069 ± 19 | (1.904± 0.003)$10^{-2}$ | 5.69 ± 0.01 | 1.739 ± 0.002 |
| 11.27± 0.01 | 94.5 ± 0.1 | 119.3 ± 0.2 | 2085 ± 19 | (1.890± 0.003)$10^{-2}$ | 5.35 ± 0.01 | 1.677 ± 0.002 |
| 11.42± 0.01 | 95.1 ± 0.1 | 120.1 ± 0.2 | 2096 ± 19 | (1.880± 0.003)$10^{-2}$ | 5.15 ± 0.01 | 1.639 ± 0.002 |
| 11.50± 0.01 | 95.5 ± 0.1 | 120.4 ± 0.2 | 2101 ± 19 | (1.875± 0.003)$10^{-2}$ | 5.07 ± 0.01 | 1.623 ± 0.002 |
| | | | unnecessary | | | |

We work out the errors for all the first row, as example.

Error for $R_B$: $\Delta R_B = R_B\sqrt{\left(\frac{\Delta V}{V}\right)^2+\left(\frac{\Delta I}{I}\right)^2} = 110.9\sqrt{\left(\frac{0.01}{9.48}\right)^2+\left(\frac{0.1}{85.5}\right)^2} = 0.2\ \Omega$

Error for $T$: $\Delta T = T\left(\frac{\Delta a}{a}+0.83\frac{\Delta R_B}{R_B}\right)$ ; $\Delta T = 1962\left(\frac{0.3}{39.4}+0.83\frac{0.2}{110.9}\right) = 18\ \mathrm{K}$

Error for $R_B^{-0.83}$ :

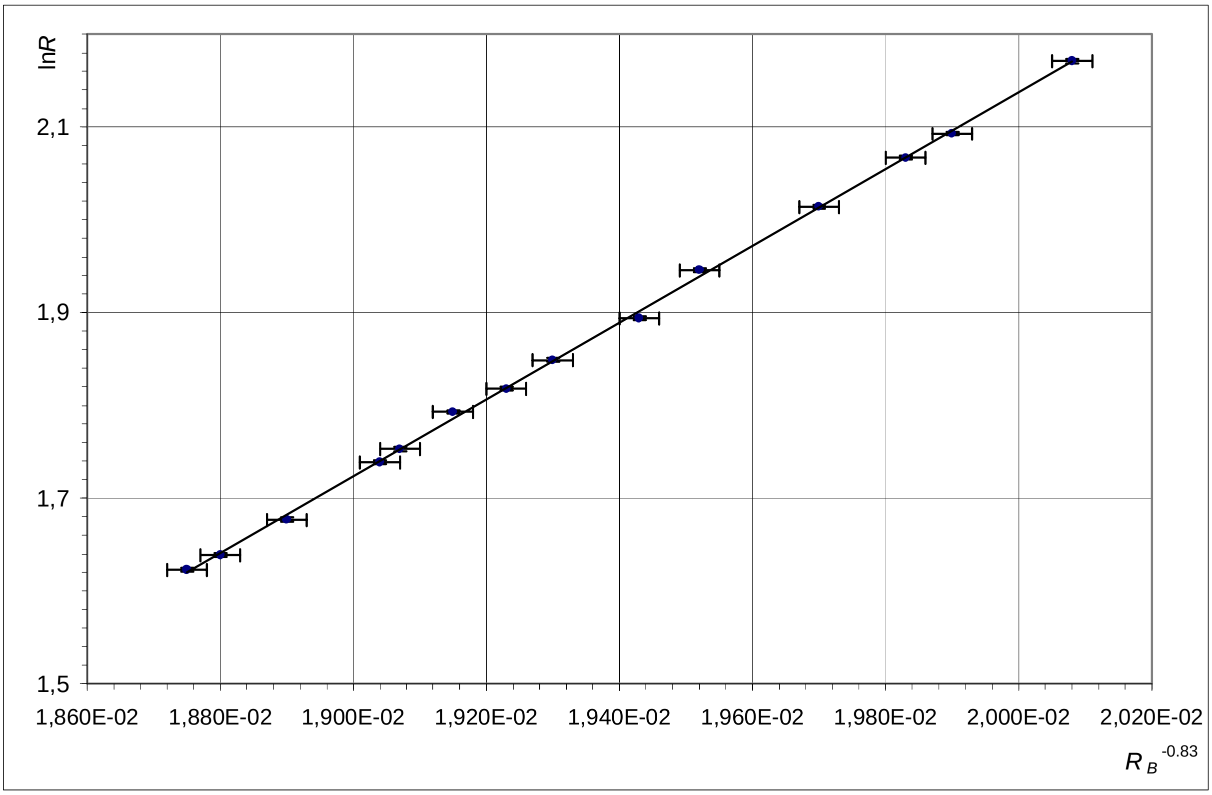
$$x = R_B^{-0.83} \ ;\ \ln x = -0.83\ln R_B \ ;\ \Delta x = x\cdot 0.83\frac{\Delta R_B}{R_B} \ ;\ \Delta\left(R_B^{-0.83}\right) = R_B^{-0.83}\,\frac{\Delta R_B}{R_B}$$

$$\Delta\left(R_B^{-0.83}\right) = 0.020077\,\frac{0.2}{110.9} \approx 0.004\times 10^{-2}$$

Error for ln$R$ : $\Delta \ln R = \frac{\Delta R}{R}$ ; $\Delta \ln R = \frac{0.01}{8.77} = 0.001$

e)

We plot ln $R$ versus $R_B^{-0.83}$ .




By the least squares

Slope $= m = 414{,}6717$

$\sum \left(R_B^{-0.83}\right)^2 = 5.23559 \times 10^{-3}$

$\sum \left(R_B^{-0.83}\right) = 0.27068$

$n = 14$

For axis $X$: $\sigma_{R_B^{-0.83}} = \sqrt{\dfrac{\sum \Delta\left(R_B^{-0.83}\right)^2}{n}} = 0.003 \times 10^{-2}$

For axis $Y$: $\sigma_{\ln R} = \sqrt{\dfrac{\sum \Delta(\ln R)^2}{n}} = 0.002$

$$\sigma = \sqrt{\sigma_{\ln R}^2 + m^2 \sigma_{R_B^{-0.83}}^2} = \sqrt{0.002^2 + 414.672^2 \cdot \left(0.003 \times 10^{-2}\right)^2} = 0.0126$$

$$\Delta m = \sqrt{\frac{n\sigma^2}{n\sum \left(R_B^{-0.83}\right)^2 - \left(\sum R_B^{-0.83}\right)^2}} = \sqrt{\frac{14 \cdot 0{,}0126^2}{14 \cdot 5.23559 \times 10^{-3} - (0.27068)^2}} = 8.295$$

Because of

$$m = \frac{c_2 \gamma}{\lambda_0 a}$$

and

$$c_2 = \frac{hc}{k}$$

then

$$h = \frac{mk\lambda_0 a}{c\gamma}$$

$$h = \frac{414.67 \cdot 1.381\times 10^{-23} \cdot 590\times 10^{-9} \cdot 39.4}{2.998\times 10^{8} \cdot 0.702} = 6.33\times 10^{-34}$$

$$\Delta h = h\sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta k}{k}\right)^2 + \left(\frac{\Delta \lambda_0}{\lambda_0}\right)^2 + \left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta \gamma}{\gamma}\right)^2}$$

$$\Delta h = 6.34\times 10^{-34}\sqrt{\left(\frac{8.3}{415}\right)^2 + 0 + \left(\frac{28}{590}\right)^2 + \left(\frac{0.3}{39.4}\right)^2 + 0 + \left(\frac{0.01}{0.70}\right)^2} = 0.34\times 10^{-34}$$

| | |
|---|---|
| $h = 6.3\times 10^{-34}$ J · s | $\Delta h = 0.3\times 10^{-34}$ J · s |