



## THEORETICAL COMPETITION

Tuesday, July 23<sup>rd</sup>, 2002

### Solution I: Ground-Penetrating Radar

1. Speed of radar signal in the material  $v_m$ :

$$\mathbf{w} - \mathbf{b}z = \text{constant} \rightarrow \mathbf{b}z = -\text{constant} + \mathbf{w} \quad (0.2 \text{ pts})$$

$$v_m = \frac{\mathbf{w}}{\mathbf{b}}$$

$$v_m = \frac{1}{\mathbf{w} \left\{ \frac{\mathbf{ne}}{2} \left[ \left( 1 + \frac{\mathbf{s}^2}{\mathbf{e}^2 \mathbf{w}^2} \right)^{1/2} + 1 \right] \right\}^{1/2}} \quad (0.4 \text{ pts})$$

$$v_m = \frac{1}{\left\{ \frac{\mathbf{ne}}{2} (1+1) \right\}^{1/2}} = \frac{1}{\sqrt{\mathbf{ne}}} \quad (0.4 \text{ pts})$$

2. The maximum depth of detection (skin depth,  $d$ ) of an object in the ground is inversely proportional to the attenuation constant:

(0.5 pts)

(0.3 pts)

(0.2 pts)

$$d = \frac{1}{a} = \frac{1}{w \left\{ \frac{\mathbf{ne}}{2} \left[ \left( 1 + \frac{\mathbf{s}^2}{\mathbf{e}^2 \mathbf{w}^2} \right)^{1/2} - 1 \right] \right\}^{1/2}} = \frac{1}{w \left\{ \frac{\mathbf{ne}}{2} \left[ \left( 1 + \frac{1}{2} \frac{\mathbf{s}^2}{\mathbf{e}^2 \mathbf{w}^2} \right) - 1 \right] \right\}^{1/2}} = \frac{1}{w \left\{ \frac{\mathbf{ne}}{2} \cdot \frac{1}{2} \frac{\mathbf{s}^2}{\mathbf{e}^2 \mathbf{w}^2} \right\}^{1/2}}$$

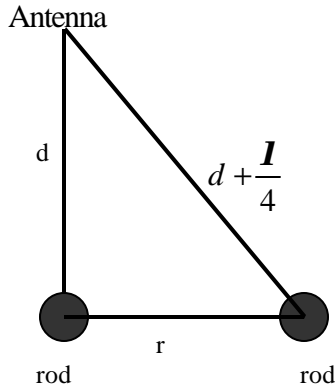
$$d = \left( \frac{2}{\mathbf{s}} \right) \left( \frac{\mathbf{e}}{\mathbf{m}} \right)^{1/2}.$$

Numerically  $d = \frac{(5.31\sqrt{\mathbf{e}_r})}{\mathbf{s}}$  m, where  $\mathbf{s}$  is in mS/m. (0.5 pts)

For a medium with conductivity of 1.0 mS/m and relative permittivity of 9, the skin depth

$$d = \frac{(5.31\sqrt{9})}{1.0} = 15.93 \text{ m} \quad (0.3 \text{ pts}) + (0.2 \text{ pts})$$

3. Lateral resolution:



$$r^2 + d^2 = \left(d + \frac{I}{4}\right)^2$$

$$r = \left(\frac{Id}{2} + \frac{I^2}{16}\right)^{1/2}$$

(1.0 pts)

$r = 0.5 \text{ m}, d = 4 \text{ m}: \frac{1}{2} = \left(\frac{4I}{2} + \frac{I^2}{16}\right)^{1/2}, I^2 + 32I - 4 = 0$  (0.5 pts)

The wavelength is  $\lambda = 0.125 \text{ m}$ .

(0.3 pts) + (0.2 pts)

The propagation speed of the signal in medium is

$$v_m = \frac{1}{\sqrt{\mathbf{m}\mathbf{e}}} = \frac{1}{\sqrt{\mathbf{m}_o\mathbf{m}_r\mathbf{e}_o\mathbf{e}_r}} = \frac{1}{\sqrt{\mathbf{m}_o\mathbf{e}_o}} \frac{1}{\sqrt{\mathbf{m}_r\mathbf{e}_r}}$$

$$v_m = \frac{c}{\sqrt{\mathbf{m}_r\mathbf{e}_r}} = \frac{0.3}{\sqrt{\mathbf{e}_r}} \text{ m/ns}, \text{ where } c = \frac{1}{\sqrt{\mathbf{m}_o\mathbf{e}_o}} \text{ and } \mathbf{m}_r = 1$$

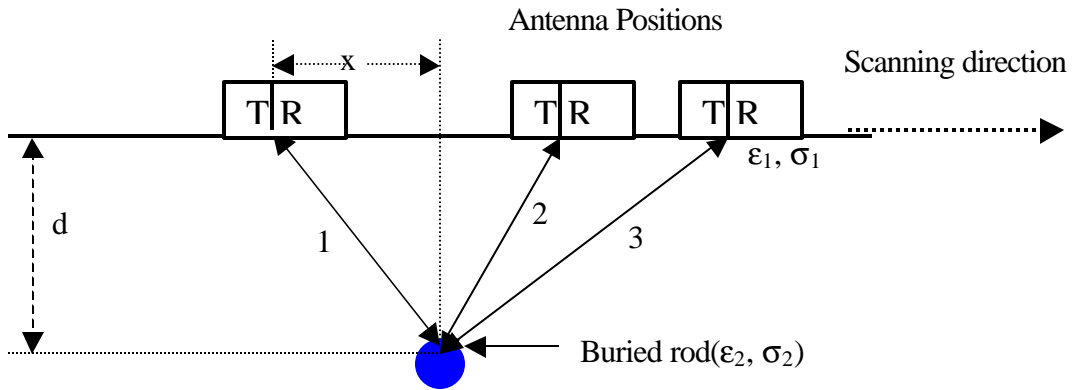
$$v_m = 0.1 \text{ m/ns} = 10^8 \text{ m/s} \quad (0.5 \text{ pts})$$

The minimum frequency need to distinguish the two rods as two separate objects is

$$f_{\min} = \frac{v}{I} \quad (0.5 \text{ pts})$$

$$f_{\min} = \frac{0.3}{0.125} \times 10^9 \text{ Hz} = 800 \text{ MHz} \quad (0.3 \text{ pts}) + (0.20 \text{ pts})$$

4. Path of EM waves for some positions on the ground surface

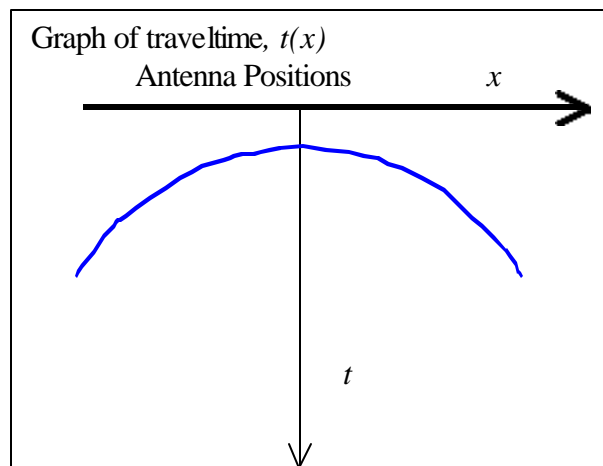


The traveltime as function of  $x$  is

$$\left(\frac{t}{2} v\right)^2 = d^2 + x^2, \quad (1.0 \text{ pts})$$

$$t(x) = \sqrt{\frac{4d^2 + 4x^2}{v}} \quad (1.0 \text{ pts})$$

$$t(x) = \frac{2\sqrt{\epsilon_{1r}}}{0.3} \sqrt{d^2 + x^2}$$



For  $x = 0$  (1.0 pts)

$$100 = 2 \times (3/0.3) d$$

$d = 5 \text{ m}$  (0.5 pts)