SOLUTION EXPERIMENT I

PART A

1. [Total 0.5 pts]
   The experimental method chosen for the calibration of the arbitrary scale is a simple pendulum method [0.3 pts]

   ![Sketch of the experimental setup]

   Figure 1. Sketch of the experimental set up [0.2 pts]

2. [Total 1.5 pts]
   The expression relating the measurable quantities: [0.5 pts]

   \[ T_{osc} = 2 \pi \sqrt{\frac{l}{g}} ; \quad T_{osc}^2 = 4 \pi^2 \frac{l}{g} \]

   Approximations:
   \[ \sin \Theta = \Theta \quad [0.5 \text{ pts}] \]
   mathematical pendulum (mass of the wire << mass of the steel ball, the radius of the steel ball << length of the wire [0.5 pts]
   flexibility of the wire, air friction, etc [0.1 pts, only when one of the two major points above is not given]
3. **[Total 1.0 pts]** Data sample from simple pendulum experiment
   
   # of cycle ≥ 20 [0.2 pts.], difference in T ≥ 0.01 s [0.4 pts], # of data ≥ 4 [0.4 pts]

<table>
<thead>
<tr>
<th>No.</th>
<th>t(s) for 50 cycles</th>
<th>Period, T (s)</th>
<th>Scale marked on the wire (arbitrary scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>91.47</td>
<td>1.83</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>89.09</td>
<td>1.78</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>86.45</td>
<td>1.73</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>83.8</td>
<td>1.68</td>
<td>50</td>
</tr>
</tbody>
</table>

4. **[Total 0.5 pts]**

<table>
<thead>
<tr>
<th>No.</th>
<th>Period, T (s)</th>
<th>Scale marked on the wire (arbitrary scale)</th>
<th>T^2(s^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.83</td>
<td>200</td>
<td>3.35</td>
</tr>
<tr>
<td>2</td>
<td>1.78</td>
<td>150</td>
<td>3.17</td>
</tr>
<tr>
<td>3</td>
<td>1.73</td>
<td>100</td>
<td>2.99</td>
</tr>
<tr>
<td>4</td>
<td>1.68</td>
<td>50</td>
<td>2.81</td>
</tr>
</tbody>
</table>

The plot of $T^2$ vs scale marked on the wire:

![Plot of $T^2$ vs scale marked on the wire](image)

5. Determination of the smallest unit of the arbitrary scale in term of mm **[Total 1.5 pts]**

\[
T_{osc1}^2 = \frac{4\pi^2}{g} L_4, \quad T_{osc2}^2 = \frac{4\pi^2}{g} L_2
\]

\[
(T_{osc1}^2 - T_{osc2}^2) = \frac{4\pi^2}{g} L_4 - L_2 = \frac{4\pi^2}{g} \Delta L
\]
\[ \Delta L = \frac{g}{4\pi^2} \left( T_{osc1}^2 - T_{osc2}^2 \right) \] or other equivalent expression \hspace{1cm} [0.5 pts]

<table>
<thead>
<tr>
<th>No.</th>
<th>Calculated ( \Delta L ) (m)</th>
<th>( \Delta L ) in arbitrary scale</th>
<th>Values of smallest unit of arbitrary scale (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.171893 ( \text{s}^2 )</td>
<td>0.042626</td>
<td>50</td>
</tr>
<tr>
<td>2.</td>
<td>0.357263 ( \text{s}^2 )</td>
<td>0.088595</td>
<td>100</td>
</tr>
<tr>
<td>3.</td>
<td>0.537728 ( \text{s}^2 )</td>
<td>0.133347</td>
<td>150</td>
</tr>
<tr>
<td>4.</td>
<td>0.18537 ( \text{s}^2 )</td>
<td>0.045968</td>
<td>50</td>
</tr>
<tr>
<td>5.</td>
<td>0.365835 ( \text{s}^2 )</td>
<td>0.09072</td>
<td>100</td>
</tr>
<tr>
<td>6.</td>
<td>0.180465 ( \text{s}^2 )</td>
<td>0.044752</td>
<td>50</td>
</tr>
</tbody>
</table>

The average value of smallest unit of arbitrary scale, \( \bar{l} = 0.89 \text{ mm} \) \hspace{1cm} [0.5 pts]

The estimated error induced by the measurement: [0.5 pts]

<table>
<thead>
<tr>
<th>No.</th>
<th>Values of smallest unit of arbitrary scale (mm)</th>
<th>( l - \bar{l} )</th>
<th>( (l - \bar{l})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.85</td>
<td>-0.04</td>
<td>0.0016</td>
</tr>
<tr>
<td>2.</td>
<td>0.89</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.</td>
<td>0.89</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4.</td>
<td>0.92</td>
<td>0.03</td>
<td>0.0009</td>
</tr>
<tr>
<td>5.</td>
<td>0.91</td>
<td>0.02</td>
<td>0.0004</td>
</tr>
<tr>
<td>6.</td>
<td>0.90</td>
<td>0.01</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

And the standard deviation is:

\[
\Delta l = \sqrt{\frac{\sum_{i=1}^{N} (l_i - \bar{l})^2}{N-1}} = \sqrt{\frac{0.003}{5}} = 0.02 \text{ mm}
\]

other legitimate methods may be used
PART B

1. The experimental set up: [Total 1.0 pts]

![Experimental Setup Diagram]

Water and electrode inside the glass tube [0.2 pts]

Electrodes [0.2 pts]

Container filled with water [0.2 pts]

2. Derivation of equation relating the quantities time \( t \), current \( I \), and water level difference \( \Delta h \): [Total 1.5 pts]

\[
I = \frac{\Delta Q}{\Delta t}
\]

From the reaction: \( 2 \text{H}^+ + 2 \text{e} \rightarrow \text{H}_2 \), the number of molecules produced in the process (\( \Delta N \)) requires the transfer of electric charge is \( \Delta Q=2e\Delta N \) : [0.2 pts]

\[
I = \frac{\Delta N \cdot 2e}{\Delta t}
\]

\[
P \Delta V = \Delta N \cdot k_B \cdot T
\]

\[
= \frac{I \Delta t}{2e} \cdot k_B \cdot T
\]

\[
P \Delta h(\pi r^2) = \frac{I \Delta t}{2} \cdot \frac{k_B}{e} \cdot T
\]

\[
I \Delta t = \frac{e \cdot 2P(\pi r^2)}{k_B \cdot T} \Delta h
\]

[0.1 pts]
3. The experimental data: [Total 1.0 pts]

<table>
<thead>
<tr>
<th>No.</th>
<th>Δh (arbitrary scale)</th>
<th>I (mA)</th>
<th>Δt (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>4.00</td>
<td>1560.41</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>4.00</td>
<td>2280.61</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>4.00</td>
<td>2940.00</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>4.00</td>
<td>3600.13</td>
</tr>
</tbody>
</table>

- The circumference φ, of the test tube = 46 arbitrary scale [0.3 pts]
- The chosen values for Δh (≥ 4 scale unit) for acceptable error due to uncertainty of the water level reading and for I (≤ 4 mA) for acceptable disturbance [0.3 pts]
- # of data ≥ 4 [0.4 pts]

The surrounding condition (T, P) in which the experimental data given above taken:
T = 300 K
P = 1.00 \times 10^5 Pa

4. Determination the value of e/k_B [Total 1.5 pts]

<table>
<thead>
<tr>
<th>No.</th>
<th>Δh (arbitrary scale)</th>
<th>Δh (mm)</th>
<th>I (mA)</th>
<th>Δt (s)</th>
<th>I Δt (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>10.68</td>
<td>4.00</td>
<td>1560.41</td>
<td>6241.64</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>14.24</td>
<td>4.00</td>
<td>2280.61</td>
<td>9120.48</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>17.80</td>
<td>4.00</td>
<td>2940.00</td>
<td>11760.00</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>21.36</td>
<td>4.00</td>
<td>3600.13</td>
<td>14400.52</td>
</tr>
</tbody>
</table>
Plot of $\Delta t$ vs $\Delta h$ from the data listed above

The slope obtained from the plot is 763.94;

$$\frac{e}{k_B} = \frac{763.94 \times 300 \times \pi}{2 \times 10^3 \times (23 \times 0.89 \times 10^{-3} \times 0.82)^2} = 1.28 \times 10^4 \text{ Coulomb K/J}$$

[1.0 pts]

Alternatively [the same credit points]

<table>
<thead>
<tr>
<th>No.</th>
<th>$\Delta h$ (mm)</th>
<th>$\Delta t$ (C)</th>
<th>Slope</th>
<th>$e/k_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.68</td>
<td>6241.64</td>
<td>584.4232</td>
<td>9774.74</td>
</tr>
<tr>
<td>2</td>
<td>14.24</td>
<td>9120.48</td>
<td>640.4831</td>
<td>10712.37</td>
</tr>
<tr>
<td>3</td>
<td>17.80</td>
<td>11760.00</td>
<td>660.6472</td>
<td>11050.07</td>
</tr>
<tr>
<td>4</td>
<td>21.36</td>
<td>14400.52</td>
<td>674.1816</td>
<td>11275.99</td>
</tr>
</tbody>
</table>
Average of $e/k_b = 1.07 \times 10^4$ Coulomb K/J

<table>
<thead>
<tr>
<th>No.</th>
<th>$e/k_b$</th>
<th>difference</th>
<th>Square difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9774.74</td>
<td>-928.55</td>
<td>862205.5</td>
</tr>
<tr>
<td>2</td>
<td>10712.37</td>
<td>9.077117</td>
<td>82.39405</td>
</tr>
<tr>
<td>3</td>
<td>11050.07</td>
<td>346.7808</td>
<td>120256.9</td>
</tr>
<tr>
<td>4</td>
<td>11275.99</td>
<td>572.6996</td>
<td>327984.9</td>
</tr>
</tbody>
</table>

Estimated error

The standard deviation obtained is $0.66 \times 10^3$ Coulomb K/J, Other legitimate measures of estimated error may be also used